1. Both the Barings’ fall and Daiwa’s huge loss involve rouge traders (Nicholas Leeson and Toshihide Igushi, respectively). Search the relevant web sites to obtain the basic facts about these two cases of poor risk management. Comment on the similarities and differences in these two cases in terms of (i) lack of controls within the institutions, (ii) how the market events triggered these huge losses.

2. Suppose that each of two investments has a 0.9% chance of a loss of $10 million, a 99.1% of a loss of $1, and 0% chance of a profit. The investments are independent of each other.

   (a) What is the VaR for one of the investments when the confidence level is 99%?
   (b) What is the expected shortfall for one of the investments when the confidence level is 99%?
   (c) What is the VaR for a portfolio consisting of the two investments when the confidence level is 99%?
   (d) What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is 99%?
   (e) Show that in this example VaR does not satisfy the subadditivity condition whereas expected shortfall does.
   (f) What is the difference between expected shortfall and VaR? What is the theoretical advantage of expected shortfall over VaR?

3. Suppose that daily gains and losses are normally distributed with standard deviation of $5 million.

   (a) Estimate the minimum regulatory capital the bank is required to hold (assume a multiplicative factor of 4.0).
   (b) Estimate the economic capital using a one-year time horizon and a 99.9% confidence limit, assuming that there is a correlation of 0.05 between gains or losses on successive days.

4. Suppose that the economic capital estimate for two business units are as follows:

<table>
<thead>
<tr>
<th>Type of risk</th>
<th>Business Unit 1</th>
<th>Business Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market risk</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Credit risk</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Operational risk</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

   The correlation between market risk and credit risk in the same business unit is 0.3. The correlation between credit risk in one business unit and credit risk in the other is 0.7. The correlation between market risk in one business unit and market risk in the other is 0.2. All other correlations are zero. Calculate the total economic capital. How much should be allocated to each business unit?
5. The return-to-maturity expectations hypothesis states that the return generated by holding a bond for term \( t \) to \( T \) will equal the expected return generated by continually rolling over a bond whose term is a period evenly divisible into \( T - t \). Explain why the above relationship can be expressed formally as

\[
\frac{1}{B(t, T)} = E_t[(1 + r_t)(1 + \tilde{r}_{t+1})\cdots(1 + \tilde{r}_{T-1})],
\]

where \( B(t, T) \) is the time-\( t \) price of a discount bond maturing at \( T \) and \( r_t \) is the one-period spot rate at time \( t \). The operator \( E_t \) indicates that expectation is taken based on the information available at the current time \( t \).

**Remark**

Suppose the investor starts with one dollar at time \( t \) and invests in a discount bond maturing one year later the “deterministic” return is

\[
1 + r_t = \frac{1}{B(t, t + 1)},
\]

where \( B(t, t+1) \) is known at time \( t \). At time \( t+1 \), the investor uses the proceed \( \frac{1}{B(t, t + 1)} \) to invest in a discount bond maturing one year later. The bond price is \( \hat{B}(t + 1, t + 2) \), which is not known at time \( t \). The return over \([t + 1, t + 2]\) is

\[
1 + \tilde{r}_{t+1} = \frac{1}{B(t + 1, t + 2)},
\]

where “tilde” quantities represent stochastic quantities. At time \( t + 2 \), the investor again invests in \( \frac{1}{B(t, t + 1)\hat{B}(t + 1, t + 2)} \) units of discount bond maturing one year later. After \( T - t \) years, the random return at time \( T \) is

\[
(1 + r_t)(1 + \tilde{r}_{t+1})\cdots(1 + \tilde{r}_{T-1}) = \frac{1}{B(t, t + 1)\hat{B}(t + 1, t + 2)\cdots\hat{B}(T - 1, T)}.
\]

This strategy is like investing in a money market account with annual rolling over.

6. Show that all curves \( r_H = r_H(i) \) for various horizons \( H(H = 1, 2, \ldots, \infty) \) go through the point \((i_0, i_0)\). In other words, show that \((i_0, i_0)\) is a fixed point for all curves \( r_H(i) \).
7. Suppose that an obligation occurring at a single time period is immunized against interest rate changes with bonds that have only nonnegative cash flows (see p.92-94 in Topic One). Let $P(\lambda)$ be the value of the resulting portfolio, including the obligation, when the interest rate is $r + \lambda$ and $r$ is the current interest rate. Here, $\lambda$ represents the change in the interest rate. By immunization construction, we have set $P(0) = 0$ and $P'(0) = 0$. In this problem, we would like to show that $P(0)$ is a local minimum; that is, $P''(0) \geq 0$.

Assume a yearly compounding convention. The discount factor at time $t$ is

$$d_t(\lambda) = (1 + r + \lambda)^{-t}.$$  

Let $d_t = d_t(0)$. For convenience, we assume that the obligation has magnitude 1 and is due at time $\tilde{t}$. The conditions for immunization are then given by

$$P(0) = \sum_t c_t d_t - d_{\tilde{t}} = 0$$

$$P'(0)(1 + r) = \sum_t t c_t d_t - \tilde{t} d_{\tilde{t}} = 0.$$

Here, the summation is taken over the various bonds in the immunized portfolio with varying maturity $t$.

(a) Show that for all values of $\alpha$ and $\beta$ there holds

$$P''(0)(1 + r)^2 = \sum_t (t^2 + \alpha t + \beta) c_t d_t - (\tilde{t}^2 + \alpha \tilde{t} + \beta)d_{\tilde{t}}.$$

(b) Since $t^2 + \alpha t + \beta$ is a quadratic equation in $t$, one can always choose $\alpha$ and $\beta$ such that the function $t^2 + \alpha t + \beta$ has a minimum at $\tilde{t}$ and has a value of 1 there. Use these results to conclude that $P''(0) \geq 0$.

8. Consider the following two bonds:
Bond | Bond B
---|---
Maturity | 15 years | 11 years
Coupon rate | 10% | 5%
Par value | $1000 | $1000

(a) The current yield to maturity is taken to be 12%. Determine the convexity of each bond.

(b) Suppose you have a defensive strategy, and that you want to immunize the investor. What is each bond’s rate of return at horizon $H = D$ if interest rates keep jumping from 12% to either 10% or 14%?

(c) By examining the rates of return of the two bonds under an increase or decrease of interest rates, and different choices of horizon, which bond would you choose?