MATH 246, Fall 2001

Midterm Test

Time allowed: 1 hour   **Course Instructor:** *Prof. Y. K. Kwok*

[points]

1. A computer manufacturer uses chips from 3 sources. Chips from sources A, B and C are defective with probabilities $0.1, 0.2$ and 0.3, respectively. If a randomly selected chip is found to be non-defective, what is the probability that the chip is *not* coming from manufacturer B?

   [4]

2. For the Poisson random variable with parameter α, show that (i) for $\alpha < 1, P[N = k]$ is maximum at $k = 0$; (ii) for $\alpha > 1, P[N = k]$ is maximum at $[\alpha]$, (the floor value of α). How to modify the result when α happens to be a positive integer?   [4]

3. (a) State the memoryless property of an exponential distribution.   [1]

   (b) Suppose that the number of hours that a light bulb can service is exponentially distributed with an average value of 10 hours. At this moment, a light bulb has been working in the past 6 hours, what is the probability that it continues to function for the next 2 hours?   [3]

4. (a) Describe the relation between the failure rate function and the reliability function.   [3]

   (b) A device has the failure rate function

   $$r(t) = \begin{cases} 1 & 0 \leq t < 10 \\ 1 + 10(t - 10) & t \geq 10 \end{cases}.$$

   Find the reliability function and the probability density function of the device.   [2]

5. The probability density function of a random variable T is given by

   $$f_T(t) = \begin{cases} \lambda e^{-\lambda(t - T_0)} & t \geq T_0 \\ 0 & t < T_0 \end{cases}.$$

   Find the probability density function of the random variable $X = 4 - 5T$.   [3]

— *End* —