MATH 246 — Probability and Random Processes

Test One

Fall 2002

Course Instructor: Prof. Y. K. Kwok

Time allowed: 75 minutes

1. Consider 4 cards whose colors on the two sides are

   black/black  red/black  red/red  black/blue.

   Suppose one card is chosen at random.
   (a) What is the probability that its upper side is black? [3]
   (b) Conditional on the occurrence that the upper side is black, what is the probability that it is the
       black/blue card? [3]

2. A dice with 6 faces is tossed two times. Let the random variable \( Y \) be the sum of the numbers shown
   in the two tosses.
   (a) Describe the sample space of \( Y, S_Y \). [2]
   (b) Find the equivalent event for the event \( \{Y = 3\} \). [2]
   (c) Find \( P[Y \leq 4] \). [2]

3. Let \( N \) be a geometric random variable with \( S_N = \{1, 2, \ldots, \} \), and let \( p \) be the probability of success in
each trial.
   (a) Find \( P[N > k] \). [1]
   (b) Find \( P[N \text{ is an even number}] \). [2]
   (c) Find \( P[N = k | N \leq m] \). Distinguish between \( k \leq m \) and \( k > m \). [3]

4. Suppose that children are born at a Poisson rate of 5.6 per day in a certain hospital.
   (a) What is the probability that at least two babies are born during the next 6 hours? You may leave
       your answer in terms of exponentials. [2]
   (b) What is the mean number of births over 2 days? [2]
   (c) What is the most possible number of births over 3 days? [2]
5. Let \( T \) be an exponential random variable with the parameter \( \lambda \), where \( T \) is used to model the lifetime of a component.

(a) Find and plot \( F_T(x|T > t) \). Is \( F_T(x|T > t) \) the same as \( F_T(x) \)? Why or why not? [3]

(b) The failure rate function \( r(t) \) is defined as \( f_T(x|T > t) \) evaluated at \( x = t \), show that

\[
r(t) = -\frac{f_T(t)}{R(t)},
\]

where \( R(t) = P[T \geq t] \) is the reliability function. [3]