MATH 246 — Probability and Random Processes

Test One

Fall 2002 **Course Instructor:** *Prof. Y. K. Kwok*

Time allowed: 75 minutes

[points]

1. Consider 4 cards whose colors on the two sides are

   black/black red/black red/red black/blue.

   Suppose one card is chosen at random.

   (a) What is the probability that its upper side is black? **[3]**

   (b) Conditional on the occurrence that the upper side is black, what is the probability that it is the black/blue card? **[3]**

2. A dice with 6 faces is tossed two times. Let the random variable Y be the sum of the numbers shown in the two tosses.

   (a) Describe the sample space of Y, S_Y. **[2]**

   (b) Find the equivalent event for the event $\{Y = 3\}$. **[2]**

   (c) Find $P[Y \leq 4]$. **[2]**

3. Let N be a geometric random variable with $S_N = \{1, 2, \cdots, \}$, and let p be the probability of success in each trial.

   (a) Find $P[N > k]$. **[1]**

   (b) Find $P[N$ is an even number$]$. **[2]**

   (c) Find $P[N = k | N \leq m]$. Distinguish between $k \leq m$ and $k > m$. **[3]**

4. Suppose that children are born at a Poisson rate of 5.6 per day in a certain hospital.

   (a) What is the probability that at least two babies are born during the next 6 hours? You may leave your answer in terms of exponentials. **[2]**

   (b) What is the mean number of births over 2 days? **[2]**

   (c) What is the most possible number of births over 3 days? **[2]**

5. Let T be an exponential random variable with the parameter λ, where T is used to model the lifetime of a component.

   (a) Find and plot $F_T(x|T > t)$. Is $F_T(x|T > t)$ the same as $F_T(x)$? Why or why not? **[3]**

   (b) The *failure rate function* $r(t)$ is defined as $f_T(x|T > t)$ evaluated at $x = t$, show that

$$r(t) = -\frac{R'(t)}{R(t)},$$

   where $R(t) = P[T \geq t]$ is the *reliability function*. **[3]**

— End —