1. Label the four cards as follows

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Black/Black</td>
<td>Red/Black</td>
<td>Red/Red</td>
<td>Black/Blue</td>
</tr>
</tbody>
</table>

Define \( C_i \) = \{the \( i \)th card is chosen\}
\( B = \{\text{the upper side is black}\} \)
(a) The required probability
\[
P[B] - \sum_{i=1}^{4} P[B|C_i]P[C_i] = (1)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + (0)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) \rightarrow -\frac{1}{2}
\]
(b) Note that \( \{C_1, C_2, C_3, C_4\} \) forms a partition of the sample space. By Bayes’s theorem, the required probability
\[
P[C_4|B] = \frac{P[B|C_4]P[C_4]}{P[B]} \rightarrow \frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}{\frac{1}{2}} \rightarrow \frac{1}{4}
\]

2. (a) \( S_Y = \{2, 3, 4, \ldots, 11, 12\} \)
(b) \( \{Y = 3\} = \{1\text{st time shown 1 and 2nd time shown 2}\} \cup \{1\text{st time shown 2 and 2nd time shown 1}\}
\( = \{(1,2),(2,1)\} \).
(c) \( P[Y = 2] - P[\{(1,1)\}] = \frac{1}{36} \)
\( P[Y = 3] - P[\{(1,2),(2,1)\}] = \frac{2}{36} \)
\( P[Y = 4] - P[\{(1,3),(2,2),(3,1)\}] = \frac{3}{36} \)
\( P[Y \leq 4] = \sum_{k=2}^{4} P[Y = k] \rightarrow \frac{1}{36} + \frac{2}{36} + \frac{3}{36} - \frac{1}{6} \)

3. Given that \( N \) is a geometric random variable with probability of success \( p \), we have
\[
P[N = k] = (1 - p)^{k-1} p.
\]
(a) \[ P[N > k] = \sum_{j=k+1}^{\infty} (1-p)^{j-1}p \]
\[ = (1-p)^k p \sum_{j=0}^{\infty} (1-p)^j \]
\[ = (1-p)^k p \cdot \frac{1}{1-(1-p)} - (1-p)^k. \]

(b) \[ P[N \text{ is an even number}] \]
\[ = \sum_{k=1}^{\infty} P[N = 2k] - \sum_{k=1}^{\infty} (1-p)^{2k-1}p \]
\[ = \frac{p}{1-p} \sum_{k=1}^{\infty} (1-p)^{2k} \]
\[ = \frac{p}{1-p} \cdot \frac{(1-P)^2}{1-(1-p)^2} \]
\[ = \frac{1-p}{2-p} \]

(c) \[ P[N - k|N \leq m] = \frac{P[N - k \cap N \leq m]}{P[N \leq m]} - \frac{P[N - k \cap N \leq m]}{1 - P[N > m]} \]
When \( k \leq m \), \( \{N - k\} \cap \{N \leq m\} = \{N - k\} \);
when \( k > m \), \( \{N - k\} \cap \{N \leq m\} = \phi \).

Hence, \( P[N - k|N \leq m] \)
\[ = \begin{cases} \frac{P[N - k]}{P[\phi]}, & k \leq m \\ \frac{(1-p)^{k-1}p}{1-(1-p)^m}, & k > m \end{cases} \]

4. Let \( N(t) \) – number of births over \( t \)-day period. Then the average number of birth over \( [0, t] \) is \( \alpha = 5.6t \) and \( P[N(t) - k] = \frac{(5.6t)^k}{k!}e^{-5.6t} \).

(a) Note that 6 hours = 0.25 day and 5.6 \times 5.6 = 0.25 = 1.4.

The required probability \( = P[N(0.25) \geq 2] \)
\[ = 1 - P[N(0.25) = 0] - P[N(0.25) = 1] \]
\[ = 1 - e^{-1.4} - \frac{1.4}{1}e^{-1.4} \]
\[ = 1 - 2.4e^{-1.4} \]
\[ = 0.4082. \]

(b) Over a 2-day period, \( \alpha = 5.6 \times 2 = 11.2 \). The mean number of births over 2 days \( = E[N(2)] = \alpha = 11.2 \).

(c) Over a 3-day period, \( \alpha = 5.6 \times 3 = 16.8 \). Since \( P[N(3) - k] \) attains its maximum at \( k = |\alpha| = 16 \), so the most possible number of births over 3 days \( = 16 \).

5. (a) Finally, we have \( F_T(x|T > t) = F_T(x - t) \), so \( F_T(x|T > t) \neq F_T(x) \) for all \( t > 0 \).
(b) First, we need to show that

\[ F_T(x | T > t) = \begin{cases} 0, & x < t \\ f_T(x) / [1 - F_T(t)], & x \geq t \end{cases} \]

Now, \( R(t) - P[T \geq t] = R(t) - 1 - F_T(t) \) and \( R'(t) - f_T(t) \);

so \( r(t) - f_T(t | T > t) = \frac{f_T(t)}{1 - F_T(t)} = \frac{-R'(t)}{R(t)} \).