



MATH 246 — Probability and Random Processes

Test One

Fall 2003

Course Instructor: Prof. Y. K. Kwok

Time allowed: 75 minutes

[points]

1. (a) Suppose events E and F are independent, show that E and F^C are also independent, where F^C is the complement of F . [3]
(b) In the experiment of drawing a card from an ordinary deck of 52 cards, let A and B be the events of getting a heart and an ace, respectively. Determine whether A and B are independent or not. Give your reasoning. [3]
2. In a trial, the judge is 65% sure that Susan has committed a crime. Person F (friend) and Person E (enemy) are two witnesses who know whether Susan is innocent or guilty.
 - Person F is Susan's friend and will lie with probability 0.25 if Susan is guilty. He will tell the truth if Susan is innocent.
 - Person E is Susan's enemy and will lie with probability 0.30 if Susan is innocent. Person E will tell the truth if Susan is guilty.What is the probability that Person F and Person E will give conflicting testimony? [6]
Hint: Let I and G be the two mutually exclusive events that Susan is innocent and guilty, respectively. Let C be the event that the two witnesses will give conflicting testimony. Find $P[C]$ based on $P[C|I]$ and $P[C|G]$.
3. Consider 4 cards whose colors on the two sides are
red/red black/blue blue/blue red/black
Suppose one card is chosen at random. Conditional on the occurrence that the upper side is black, what is the probability that it is the black/blue card? [6]
4. Consider a square of unit side placed in the x - y plane with corners at $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$. A point (x, y) is chosen at random inside the square. Let Z be the random variable that gives the sum of the two co-ordinates.
 - (a) Let $F_Z(z)$ denote the cdf of Z , find $F_Z(0)$, $F_Z(1)$ and $F_Z(2)$. [3]
 - (b) Let $f_Z(z)$ denote the pdf of Z , find $f_Z(-1)$ and $f_Z(3)$. [3]
5. Assume that telephone call-ins arrive at the rate of 8 calls per minute.
 - (a) Find the probability that at least one call comes in over t minutes. [3]
 - (b) Find the probability that two calls occur over a period of 30 seconds. Express your answer in terms of exponentials. [3]

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