MATH 246 — Probability and Random Processes

Solution to Test One

Fall 2003 **Course Instructor:** *Prof. Y. K. Kwok*

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1. (a) $P[E] = P[E \cap F] + P[E \cap F^C] = P[E]P[F] + P[E \cap F^C]$
   so that $P[E \cap F^C] = (1 - P[F])P[E] = P[E]P[F^C]$

   (b) $P[A] = \frac{1}{4}, P[B] = \frac{1}{13}, P[A \cap B] = \frac{1}{52}$.
   Since $P[A \cap B] = P[A]P[B]$, so they are independent.

2. By the law of total probabilities

$$\begin{aligned} P[C] &= P[C|I]P[I] + P[C|G]P[G] \\ &= 0.30 \times 0.35 + 0.25 \times 0.65 = 0.2675. \end{aligned}$$

3. $P[II|B] = \dfrac{P[II \cap B]}{P[B]} = \dfrac{\frac{1}{4} \times \frac{1}{2}}{\frac{1}{4} \times 0 + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times 0 + \frac{1}{4} \times \frac{1}{2}} = \dfrac{1}{2}$.

4. (a) Range of $S_Z = \{z : 0 \leq z \leq 2\}$.

$$\begin{aligned} F_Z(0) &= P[Z \leq 0] = 0, \\ F_Z(1) &= P[Z \leq 1] = \frac{\text{area of half square bounded below by } x = y}{\text{area of the square}} = \frac{1}{2}, \\ F_Z(2) &= 1. \end{aligned}$$

   (b) Since $F_Z(z) = 0$ for $z \leq 0$ and $F_Z(z) = 1$ for $z \geq 2$, so $f_Z(-1) = 0$ and $f_Z(3) = 0$.

5. (a) Average number of call over t minutes $= 8t$.
   $P[\text{at least one call}] = 1 - P[\text{no call}] = 1 - e^{-8t}$.

   (b) $P[X = 2] = \dfrac{4^2}{2!}e^{-4}$,
   where $\mu =$ average number of calls over half a minute $= 4$.