MA246
Homework One

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1. (a) Let A and B be mutually exclusive events such that $P[A]P[B] > 0$. Show that these events are not independent.
   (b) Let A and B be independent events such that $P[A]P[B] > 0$. Show that these events are not mutually exclusive.
   (c) Show that the impossible event is independent of every other event.

2. Two fair dice are thrown. Let E_7 denote the event that the sum of the dice is 7. Let F denote the event that the first die equals 4 and let T denote the event that the second die equals 3. Show that
   (a) E_7 is independent of F and T,
   (b) E_7 is not independent of $F \cap T$.

3. A chemical engineer is in charge of a particular process at an oil refinery. Past experience indicates that 10% of all shutdowns are due to equipment failure *alone*, 5% are due to a combination of equipment failure and operator error, and 40% involve operator error. A shutdown occurs. Find the probability that
   (a) equipment failure or operator error is involved
   (b) operator error alone is involved
   (c) neither operator error nor equipment failure is involved
   (d) operator error is involved given that equipment failure occurs
   (e) operator error is involved given that equipment failure does not occur

   Are the events E: an operator error occurs, and F: an equipment failure occurs independent? Explain.

4. A plane is missing and it is presumed that it was equally likely to have gone down in any of 3 possible regions. Let $1 - \alpha_i$ denote the probability the plane will be found upon a search of the ith region when the plane is, in fact, in that region, $i = 1, 2, 3$. (The constants α_i are called *overlook probabilities* because they represent the probability of overlooking the plane; they are generally attributable to the geographical and environmental conditions of the regions.) What is the conditional probability that the plane is in the ith region, given that a search of region 1 is unsuccessful, $i = 1, 2, 3$?

5. Suppose that an insurance company classifies people into one of three classes — good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, $0.05, 0.15$, and 0.30. If 20 percent of the population are "good risks," 50 percent are "average risks," and 30 percent are "bad risks," what proportion of people have accidents in a fixed year? If policy holder A had no accidents in 1999, what is the probability that he or she is a good (average) risk?

6. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed, and the other two are to be freed. Prisoner A asks the jailer to tell him privately one of his fellow prisoners that will be set free, claiming that there would be no harm in divulging this information because he already knows that at least one of the two will go free. The jailer refuses to answer this question, pointing out that if A knew which of his fellow prisoners were to be set free, then his own probability of being executed would rise from $\frac{1}{3}$ to $\frac{1}{2}$ because he would then be one of two prisoners. What do you think of the jailer's reasoning?

   Hint: Let E_A denote the event that Prisoner A will be executed, and similar definition for E_B and E_C. Let $E_{\widetilde{C}}$ denote the event that Prisoner C will be set free. The question asks whether

   $$P[E_A|E_{\widetilde{C}}] = P[E_A]?$$

   Note that $P[E_{\widetilde{C}}|E_A] = 1/2$ since the jailer chooses one person among Prisoners B and C to set free.

7. A coin is tossed n times. Let the random variable Y be the difference between the number of heads and the number of tails.
   a. Describe the sample space of Y, S_Y.
   b. Find the equivalent event for the event $\{Y = 0\}$.
   c. Find the equivalent event for the event $\{Y \leq k\}$ for k a positive integer.

8. A dart is thrown onto a square b units wide. Assume that the dart is equally likely to fall anywhere in the square. Let the random variable Z be given by the sum of the two coordinates of the point where the dart lands.
   a. Describe the sample space of Z, S_Z.
   b. Find the region in the square corresponding to the event $\{Z \leq z\}$ for $-\infty < z < \infty$.
   c. Find $P[Z \leq z]$.