MATH 246 — Probability and Random Processes

Test One

Fall 2004   **Course Instructor:** *Prof. Y. K. Kwok*

Time allowed: 50 minutes

[points]

1. An **unfair** coin is tossed three times. Define the events

$$A = \{\text{first toss is head}\}$$
$$B = \{\text{exactly two heads are tossed in consecutive tosses}\}$$

   Are the above events independent? Give details of your justification. **[4]**

   Hint: The probability of getting a head is *not* the same as that of getting a tail.

2. If A and B are events having positive probability. State whether each of the following statement is (i) necessarily true, (ii) necessarily false, or (iii) possibly true. Give your explanation in details.
   (a) If A and B are independent, then they are mutually exclusive. **[2]**
   (b) $P[A] = P[B] = 0.6$, and A and B are mutually exclusive. **[2]**
   (c) $P[A] = P[B] = 0.6$, and A and B are independent. **[2]**

3. A box contains 5 red and 5 blue balls. One ball is selected at random and is discarded without its colour being seen. If a second ball is drawn at random and observe to be red, what is the probability that the first discarded ball was red? **[8]**

4. Consider a square of unit side in the $x - y$ plane with corners at $(0,0), (0,1), (1,0)$ and $(1,1)$. A point (x, y) is chosen at random inside the square. Let Z be the random variable that gives the difference $x - y$ of the two coordinates.
   (a) Find the range of Z, S_Z. **[3]**
   (b) Let $F_Z(z)$ denote the cdf of Z, find $F_Z(0)$ and $F_Z(100)$. **[4]**