MATH 246 — Probability and Random Processes

Solution to Test One

Fall 2004 **Course Instructor:** *Prof. Y. K. Kwok*

---

1. Let p be the probability of getting a head, where $p \neq 0.5$. The events are

$$\begin{aligned} A &= \{HHH, HHT, HTH, HTT\} \\ B &= \{HHT, THH\} \\ A \cap B &= \{HHT\} \\ P[A] &= p, P[B] = 2p^2(1-p), \quad P[A\cap B] = p^2(1-p) \end{aligned}$$

Note that $P[A \cap B] \neq P[A]P[B]$ since $p \neq 0.5$, hence A and B are not independent.

2. (a) Necessarily false since if A and B are mutually exclusive then

$$0 = P[A \cap B] \neq P[A]P[B].$$

(b) Necessarily false since if A and B are mutually exclusive then

$$P[A \cup B] = P[A] + P[B] = 1.2 > 1$$

and this is impossible.

(c) It can be possibly true since

$$P[A]P[B] = 0.36$$

and it is possible that $P[A \cap B]$ assumes the value 0.36.

3. Let B and R be the events that the discarded ball is blue and red, respectively, and let $\widehat{R}$ be the event that the second ball is red. Now, $\{B, R\}$ is a partition of the sample space. By Bayes' Theorem, we have

$$P[R|\widehat{R}] = \frac{R[\widehat{R}|R]P[R]}{P[\widehat{R}|R]P[R] + P[\widehat{R}|B]P[B]}.$$

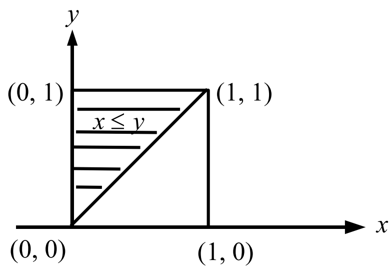
Now, $P[R] = P[B] = \dfrac{1}{2}$ and $P[\widehat{R}|R] = \dfrac{4}{9}$ and $P[\widehat{R}|B] = \dfrac{5}{9}$ so that

$$P[R|\widehat{R}] = \frac{\frac{1}{2} \times \frac{4}{9}}{\frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{5}{9}} = \frac{4}{9}.$$

4. (a) Since $0 \leq x \leq 1$ and $0 \leq y \leq 1$ so that $-1 \leq x - y \leq 1$. Hence

$$S_Z = \{z : -1 \leq z \leq 1\}.$$

(b) $$\begin{aligned} F_Z(0) &= P[Z \leq 0] = P[x \leq y] \\ &= \frac{\text{area of shaded region}}{\text{area of square}} \\ &= \frac{1}{2}. \end{aligned}$$



$F_Z(100) = P[Z \leq 100] = 1$. This is because $\{Z \leq 100\}$ is a sure event as $-1 \leq Z \leq 1$.