MATH 246 — Probability and Random Processes

Test Two

Fall 2004   **Course Instructor:** *Prof. Y. K. Kwok*

Time allowed: 50 minutes

[points]

1. At an intersection, there are two accidents per day on average. Assume that the inter-accident times follow the exponential distribution. What is the probability that after the next accident there will be no accidents at all for the next two days? Leave your answer in exponential.

    [4]

2. Let X be a Poisson random variable with parameter α. Show that $P[X = k]$ increases monotonically and then decreases monotonically as k increases, reaching its maximum value when k is the largest integer not exceeding α. What happens when α is an integer? **[7]**

    Hint: Consider the separate cases (i) $\alpha < 1$, (ii) $\alpha \geq 1$.

3. If X is a geometric random variable with probability of failure q, show analytically that

    $$P[X = k + j | X > j] = P[X = k].$$

    Give a verbal argument using the nature of a geometric random variable as to why the above relation holds. **[6]**

    Hint: $P[X = k] = pq^{k-1}$, where $p = 1 - q$.

4. If the continuous random variable X has the following pdf

    $$f_X(x) = \begin{cases} \frac{2}{9}(x + 1) & \text{for } -1 < x < 2 \\ 0 & \text{otherwise} \end{cases}.$$

    Find the pdf of $Y = X^2$. **[8]**

    Hint: The transformation function $y = x^2$ is not monotonic in $(-1, 2)$. It is necessary to divide the interval $(-1, 2)$ into 2 parts: $(-1, 1)$ and $[1, 2)$. Give the functional form of $f_Y(y)$ for $y \in (0, 1), y \in [1, 4)$ and $y \notin (0, 4)$.