# MATH 246, Fall 1999

## Midterm Test

*Time allowed: 1 hour* Instructor: Dr. Y. K. Kwok

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**[points]**

1. (a) Show that if $E \subset F$, then $P(E) \leq P(F)$.
   *Hint*: Write $F$ as the union of two mutually exclusive events, one of them being $E$. **[2]**
   (b) Show that for any two events $E$ and $F$ **[2]**
   $$P[E \cap F] \geq P[E] + P[F] - 1.$$
   (c) If $E$ and $F$ are independent events, show that $E$ and $F^C$ are also independent events. **[2]**

2. Suppose that there was a cancer diagnostic test that was 95 percent accurate both on those that do and those that do not have the disease. If 0.4 percent of the population have cancer, compute the probability that a tested person has cancer, given that his or her test result indicates so. **[4]**
   *Remark*: Just write down the arithmetic expression for the required probability. It is not necessary to evaluate it numerically.

3. A random variable $X$ has probability density function
   $$f_X(x) = \begin{cases} cx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}.$$
   (a) Find the value $c$, and compute $P\left[\frac{1}{2} \leq X \leq \frac{3}{4}\right]$.
   *Hint:* $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$ **[2]**
   (b) Find the cumulative density function $F_X(x)$, and sketch the plot of $F_X(x)$ against $x$. **[2]**

4. Let $N$ be a geometric random variable with $S_N = \{1, 2, \cdots\}$, where
   $$P[N = k] = (1-p)^{k-1}p, \quad k = 1, 2, \cdots$$
   (a) Find $P[N = k | N \leq m]$, for (i) $1 \leq k \leq m$, (ii) $k > m$. **[3]**
   (b) State and prove the memoryless property of the geometric random variable, and give the probabilistic interpretation of the property. **[3]**

— *End* —