1. If \( f(z) = 1/z = u + iv \), construct several members of the families: \( u(x, y) = \alpha, v(x, y) = \beta \) where \( \alpha \) and \( \beta \) are non-zero constants, showing that they are families of circle.

2. For each of the following functions, examine whether the function is continuous at \( z = 0 \):
   
   (a) \( f(z) = \begin{cases} 0 & z = 0 \\ \Re z & z \neq 0 \end{cases} \)
   
   (b) \( f(z) = \begin{cases} 0 & z = 0 \\ \frac{\Re z^2}{|z|} & z \neq 0 \end{cases} \)

3. A particle moves along a curve \( z = e^{-t}(2 \sin t + i \cos t) \).
   
   (a) Find a unit tangent vector to the curve at the point where \( t = \pi/4 \).
   
   (b) Determine the magnitudes of velocity and acceleration of the particle at \( t = 0 \) and \( \pi/2 \).

4. Consider the function \( f(z) = xy^2 + ix^2y, z = x + iy \). Find the point set where
   
   (a) the Cauchy-Riemann relations are satisfied;
   
   (b) the function is differentiable;
   
   (c) the function is analytic.

5. Let \( f(z) \) be analytic in a domain \( D \). Suppose \( \Re f(z) = [\Im f(z)]^2 \) inside \( D \), show that \( f(z) \) is constant inside \( D \).

6. Find an analytic function \( f(z) \) whose real part \( u(x, y) \) is
   
   (a) \( u(x, y) = y^3 - 3x^2y, \quad f(i) = 1 + i \);
   
   (b) \( u(x, y) = \frac{y}{x^2 + y^2}, \quad f(1) = 0 \);
   
   (c) \( u(x, y) = (x - y)(x^2 + 4xy + y^2) \).

7. Find the orthogonal trajectories of the following families of curves:
   
   (a) \( x^3y - xy^3 = \alpha \);
   
   (b) \( 2e^{-x} \sin y + x^2 - y^2 = \alpha \);
   
   (c) \( (r^2 + 1) \cos \theta = \alpha r \).

8. Let \( \theta = \angle APB \), which is the angle included between the line segments \( PA \) and \( PA \). Here, \( A \) and \( B \) are the fixed points \((-a, 0) \) and \( (a, 0) \), respectively, and \( P \) is the variable point \( z = x + iy \). Show that \( \theta(x, y) \) is a harmonic function. Find the corresponding harmonic conjugate \( v \) such that \( \theta + iv \) is an analytic function.
9. If $u$ and $v$ are harmonic in a region $\mathcal{R}$, prove that

$$
\left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + i \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
$$

is analytic in $\mathcal{R}$.

10. Suppose the isothermal lines of a steady state temperature field are the family of curves

$$
x^2 + y^2 = \alpha, \quad \alpha > 0.
$$

Find the general solution of the temperature function, and the equation of the family of flux lines.