MATH304

Homework 3

Course Instructor: *Prof. Y. K. Kwok*

1. Find all the values of (a) $\cosh^{-1} i$, (b) $\sinh^{-1}\{\log(-1)\}$, (c) $\tan^{-1}(2i)$, (d) $\tanh^{-1} 2$.

2. Show that $\text{Re}\,\{\sin^{-1} z\} = \arcsin \dfrac{1}{2}\{\sqrt{x^2 + y^2 + 2x + 1} - \sqrt{x^2 + y^2 - 2x + 1}\}$.

3. Show that
$$\left(\frac{ia - 1}{ia + 1}\right)^{ib} = \exp(-2b \cot^{-1} a), \quad \text{where } a \text{ and } b \text{ are real.}$$

4. Show that the mapping function
$$w = \cosh z$$
   maps the semi-infinite strip $\left\{z = x + iy : x \geq 0 \text{ and } 0 \leq y \leq \dfrac{\pi}{2}\right\}$ in the z-plane onto the first quadrant of the w-plane.

5. Let $w^3 = z$ and suppose that corresponding to $z = 1$ we have $w = 1$. (a) If we start at $z = 1$ in the z plane and make one complete circuit counterclockwise around the origin, find the value of w on returning to $z = 1$ for the first time. (b) What are the values of w on returning to $z = 1$ after $2, 3, 4, \cdots$ complete circuits about the origin? Discuss (a) and (b) if the paths do not enclose the origin.

6. Find $\dfrac{d}{dz} z^{2/3}$ at $z = -8i$ when the principal branch is used.

7. Consider a branch of $z^{1/2}$ that is analytic in the domain consisting of the z-plane less the points on the branch cut $y = 0, x \leq 0$. Suppose we choose the branch where $z^{1/2} = 2$ when $z = 4$. What value does this branch assume when

$$z = 9\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)?$$