1. Evaluate \( \int (z^2 + 3z) \, dz \) along the circle \(|z| = 2\) from \((2,0)\) to \((0,2)\) in a counterclockwise direction.

2. Evaluate \( \oint_C z^2 \, dz \) around the circles (a) \(|z| = 1\), (b) \(|z - 1| = 1\).

3. Let \( C \) be any simple closed curve bounding a region having area \( A \). Prove that

\[
A = \frac{1}{2} \oint_C x \, dy - y \, dx.
\]

Use the result to find the area bounded by the ellipse: \( x = a \cos \theta, y = b \sin \theta, 0 \leq \theta < 2\pi \).

4. Let \( C \) be the arc of the circle \(|z| = 2\) from \( z = 2 \) to \( z = 2i \) that lies in the first quadrant.

Without evaluating the integral, show that

\[
\left| \int_C \frac{1}{z^2 - 1} \, dz \right| \leq \frac{\pi}{3}.
\]

5. Let \( C \) represent a semi-circle of radius \( R \), with center at the origin, where \( R > 1 \), and consider the functions

\[
f_1(z) = z^2 - 1, \quad f_2(z) = \frac{1}{z^2 + 1}.
\]

(a) Using the triangle inequalities, show that when \( z \) assumes values on \( C \)

\[
R^2 - 1 \leq |f_1(z)| \leq R^2 + 1, \quad \frac{1}{R^2 + 1} \leq |f_2(z)| \leq \frac{1}{R^2 - 1}.
\]

(b) Deduce that

\[
\left| \int_C f_1(z) \, dz \right| \leq \pi R(R^2 + 1), \quad \left| \int_C f_2(z) \, dz \right| \leq \frac{\pi R}{R^2 - 1}.
\]

(c) Hence show that

\[
\left| \int_C f_1(z) f_2(z) \, dz \right| \leq \pi R \frac{R^2 + 1}{R^2 - 1}.
\]
6. Let $C$ be the circle: $|z| = r > 1$.
Evaluate
\[ \oint_C \frac{e^z}{(z^2 + 1)^2} \, dz. \]

7. By evaluating $\oint_C e^z \, dz$ around the circle $|z| = 1$, show that
\[ \int_0^{2\pi} e^{\cos\theta} \cos(\theta + \sin\theta) \, d\theta = \int_0^{2\pi} e^{\cos\theta} \sin(\theta + \sin\theta) \, d\theta = 0. \]

8. Let
\[ g(z) = \oint_{|\zeta| = 2} \frac{2\zeta^2 - \zeta + 1}{\zeta - z} \, d\zeta. \]
Compute (a) $g(1)$; (b) $g(z_0), |z_0| > 2$. Can we evaluate $g(2)$?

9. If $f(z)$ is an $n$th-degree polynomial with non-zero leading coefficient,
\[ f(z) = a_0z^n + a_1z^{n-1} + \cdots + a_n, \]
and $C$ is a simple closed contour enclosing all the zeros of $f(z)$, show that
\[ \frac{1}{2\pi i} \oint_C \frac{zf'(z)}{f(z)} \, dz = -\frac{a_1}{a_0}. \]

10. (a) Let $f(z)$ be analytic inside and on a simple closed curve $C$. Prove that if $f(z) \neq 0$ inside $C$, then $|f(z)|$ must assume its minimum value on $C$.
(b) Give an example to show that if $f(z)$ is analytic inside and on a simple closed curve $C$ and $f(z) = 0$ at some point inside $C$, then $|f(z)|$ need not assume its minimum value on $C$. 