AMERICAN CURRENCY FORWARD

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The usual forward contract specifies the exact maturity date for the delivery of a predetermined fixed amount of the underlying asset. In order to make the forward more attractive to potential customers, the forward writer may structure a contract that allows certain flexibilities on the period of delivery, frequency of deliveries and the amount in each delivery. Recently, a new form of American style currency forward contract has gained popularity in the over-the-counter foreign exchange markets. The holder of the American currency forward agrees to exchange a notional amount of domestic currency into foreign currency by the maturity date of the contract. However, she is given the right to exchange *any* portion of the notional at *any* time during the life of the contract, according to the exchange rates that have been pre-specified over different periods within the contract's life. Note that there are no limitations on the total number of deliveries and the amount in each delivery, except that the whole notional amount of domestic currency must be converted into foreign currency by the maturity date.

At the first glance, the vested flexibility of converting any amount of domestic currency into foreign currency at any time within the contract's life seems to be highly beneficial to the holder. The corresponding optimal conversion policy to be adopted by the holder would seem quite complicated. However, it can be argued that if the holder chooses to convert at any instant, she should optimally convert the whole notional amount (or otherwise not to convert any amount at all). To understand the argument, observing that with known values for the time to expiry and level of prevailing exchange rate, one can determine both the *exercise value* of a given amount of domestic currency if that amount is converted into foreign currency immediately and the *continuation value* if no conversion occurs. The holder will choose to exercise optimally whenever the exercise value exceeds the continuation value. Importantly, this exercise (or conversion) decision is *independent* of the amount to be exercised. Therefore, when the prevailing exchange rate reaches certain value beyond which the exercise value exceeds the continuation value, then the holder should optimally exercise the whole notional amount in one whole sum. It would be sub-

optimal to leave any amount unexercised since the continuation value of the unexercised portion is below the corresponding exercise value upon conversion.

Once the optimal policy of either converting all or none is revealed, the analysis of the American currency forward resembles that of the American currency call option. The only difference is that the exercised payoff is $S - K$ for a forward while the payoff becomes $\max(S - K, 0)$ for a call option. Here, S is the exchange rate and K is the forward's delivery price (or the option's strike price). To price an American currency forward in the Black-Scholes framework, one may apply the standard trinomial scheme, incorporating the usual dynamic programming procedure that takes the maximum of the continuation value and exercise value at each lattice node.

It would be interesting to compare the optimal exercise boundary and the early exercise premium of an American currency forward with those of the American currency call option counterpart. In FIGURE A, we show the optimal exercise boundary $S^*(\tau)$ of an American currency forward and the call option counterpart, where τ is the time to expiry. We assume constant delivery price of 50 throughout the forward's life, and the same value is used for the option's strike price. In the calculations, we assume the values of volatility, domestic interest rate and foreign interest rate to be 30%, 10% and 8%, respectively. The figure reveals that $S^*_{forward} < S^*_{option}$ at all values of τ. This result indicates that the currency forward holder optimally chooses to exercise at a lower critical exchange rate in comparison with the currency option holder. To explain this phenomenon using financial argument, we recall the advantages and disadvantages of the early exercise decision. Upon early exercising, for both forward and call option, the holder gains on the foreign interest payments received by holding the foreign currency but loses on the domestic interest payments. In addition, the option holder also loses on the intrinsic insurance value associated with the option (the protection against downside drop in exchange rate). Hence, the option holder should choose to exercise at a higher critical exchange rate so that the chance of regret of exchange rate dropping at future times is small.

In FIGURE B, we show the comparison of the ratio of the early exercise premium to the European value (in percentage) of an American currency forward and the call option counterpart. The same set of parameter values is adopted as those in FIGURE A. In addition, the exchange rate and the time to expiry are taken to be 50 and 2, respectively. For both the forward and call option, we observe that the ratio of the early exercise premium to the value of the European counterpart increases with volatility, and the rate of increase is more significant for the forward. The reasons

for these observations are two-fold. On one hand, the early exercise right is more valuable to the forward holder since the net gain upon early exercising is higher compared to that of the call option. On the other hand, the value of a European forward stays constant for all volatilities while the value of a European call option increases with increasing volatility. This leads to a smaller ratio resulted for the American call option.

Further issues to be explored on the pricing of the American currency forward should include the analysis on the impact of the stochastic domestic and foreign interest rates on the early exercise decision and the early exercise premium. Also, under stochastic interest rates environment, how the forward writer sets the delivery prices over different periods of the forward's life?

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FIGURE A: COMPARISON OF THE OPTIMAL EXERCISE BOUNDARY OF AN AMERICAN CURRENCY FORWARD AND ITS AMERICAN CURRENCY CALL OPTION COUNTERPART.
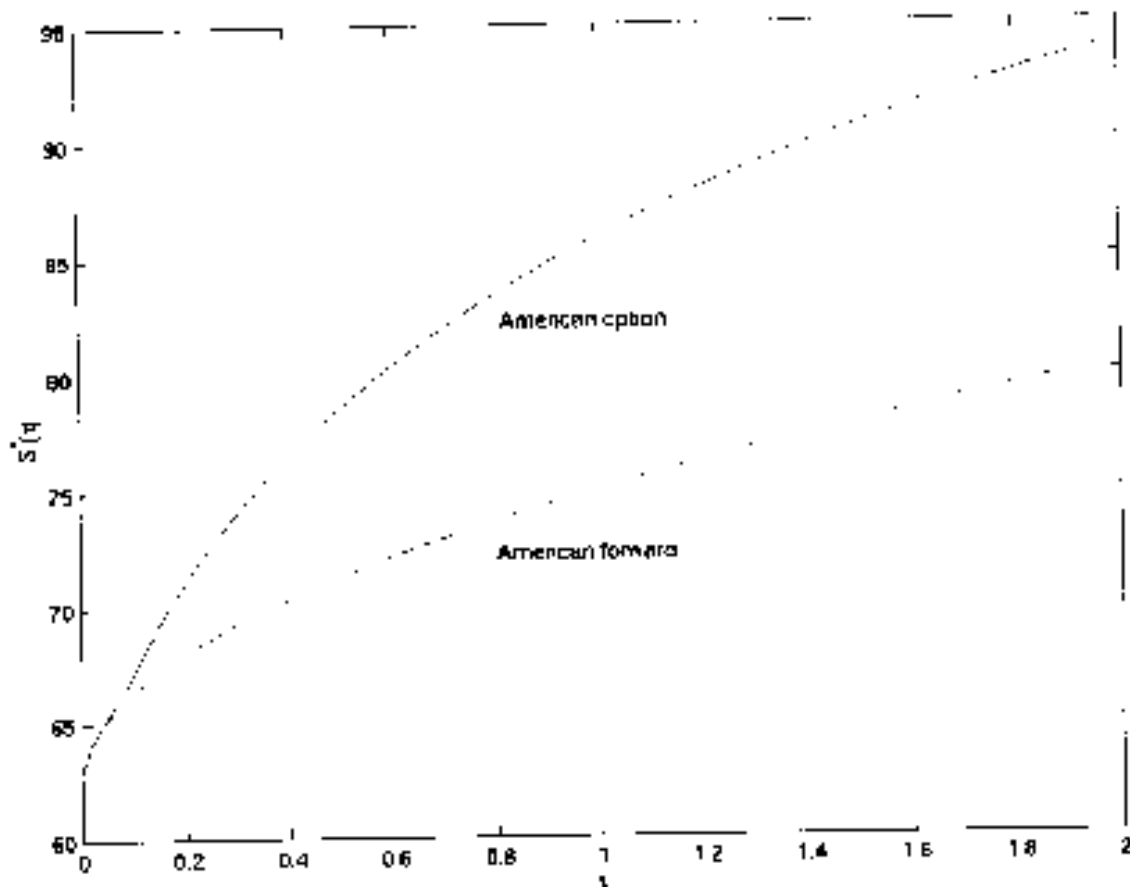
FIGURE B: COMPARISON OF THE RATIO OF THE EARLY EXERCISE PREMIUM TO THE EUROPEAN VALUE (IN PERCENTAGE) OF AN AMERICAN CURRENCY FORWARD AND ITS AMERICAN CURRENCY CALL OPTION COUNTERPART.