Employee stock option valuation with repricing features

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Abstract

Repricing of an employee stock option refers to the practice of lowering the strike price and /or extending the maturity date of a previously granted employee stock option. Normally, firms reprice after a period of significant stock price decline that renders the employee stock options deeply out-of-the-money. By modeling various repricing mechanisms based on some form of Brownian functional of the stock price process, we investigate the impact of the embedded repricing flexibility on the market value of the employee stock options. We manage to derive analytic representation of the price functions of the repriceable options. We also construct the lattice tree type option valuation algorithms by applying the forward shooting grid technique to incorporate the path dependent feature of the Brownian functional in the repriceable option models. Our calculations show that the repricing flexibility may have varying degrees of impact on the option values and their comparative statics. The option delta (option vega) values of the repriceable options are seen to be lower (higher) than those of the vanilla options.

1 Introduction

Employee stock options are considered as a standard component of the compensation package used to attract, retain and motivate employees. The value of this equity compensation may move up and down, depending on the performance of the firm's stock price. In the event of falling stock price and the options become underwater (out-of-the-money), the employees are usually the first group of people that feel the loss of wealth. In order to avoid the disappointed employees from leaving the firms, many firms are tempted to lower the strike price and / or extend the maturity date of the employee options when the firm's stock price falls substantially. In the sample of firms analyzed by Chance *et al*. (2000), they find that repricing usually follows a period of about one year of poor firm-specific performance in which the average firm loses one-fourth of its value. Besides the traditional repricing, the 6 & 1 (six-month-one-day) repricing has become more popular recently. In this so-called synthetic repricing, the firm cancels the underwater options and replaces them by new options that are received six months and one day later at strike equals the then-current stock price. Compared to the firms that continued the traditional repricing over the same time period, 6 & 1 repricing firms are shown to have higher growth potential and are more likely to be followed by analysts (Zheng, 2004).

Repricing is considered as an embedded flexibility to the employer. According to the Towers Perrin survey report in late 1990s, 44% of the firms responding to the survey claim that repricing is not allowed under the terms of the stock option plan. Chen (2004) investigates the determinants of firms' repricing policies. He discovers that firms that have better internal governance, ability to use more powerful stock-based incentives, or have less shareholder scrutiny are more likely to retain repricing flexibility. Ferri (2004) shows from his analysis of over 4,000 firm-level repricings of executives and employees' stock options that there is a significant evidence of managerial self-serving behavior in the design of the repricing offer. He finds that repricings are timed just before significant price increase when the CEO participates in the repricing, but not in employee-only repricings.

The practice of repricing has been controversial. Such practice has drawn criticism for weakening managerial incentives. With the embedded repricing flexibility, the employees may perceive that they do not need to care too much about overall stock price performance since their vested value in the options will be relatively well protected. However, when options become deep underwater, firms face the pressure to reprice in order to address employee incentive concerns. Repricing is a valuable retention tool and indirectly helps retain shareholder value. Based on their two-step utility model, Acharya *et al*. (2000) find that though the anticipation of repricing weakens incentives in the original option award, the repricing flexibility can be a value-enhancing strategy for firms to use, even in ex-ante sense. They show that repricing implies two competing effects, a negative feedback on initial incentives but a positive incentive effect that gives the principal greater ability to influence continuation outcomes. Chidambaran and Prabhala (2003) argue that repricing costs are modest, and the benefits flow mostly to non-executive employees. Also, restriction of repricing may simply force firms to other inferior contractual choices and create deadweight losses to shareholders. Carter and Lynch (2003) find little evidence that repricing affects executive turnover. Their study indicates that repricing helps lower turnover due to underwater options.

There have been several earlier papers that examine the impact of repricing on the market value of employee stock options. Using the Black-Scholes pricing framework and assuming repricing to be triggered once the stock price touches the preset trigger threshold, Brenner *et al*. (2000) and Johnson and Tian (2000) model a repriceable employee stock option as the sum of two barrier options: a down-and-out barrier call with one-touch knock-out barrier and a down-and-in barrier call with the same barrier but a new reset value of strike price. Corrado *et al*. (2001) use the utility maximization approach for pricing options that allow multiple times of repricing.

In this paper, we would like to examine how the embedded repricing flexibility may affect the ex-ante option value. Instead of employing the one-touch mechanism to trigger repricing, we follow the Carr-Linetsky's Brownian functional (2000) approach by assuming that the firm exercises repricing only when the stock price falls below some target barrier level for a certain period of time. The time duration of the stock price process staying below the barrier can be measured by either the excursion time or occupation time of the stock price path below the barrier. Compared to the one touch mechanism, the Brownian functional approach reflects better the reality of the stock price movement before repricing occurs.

The paper is organized as follows. In the next section, we state the model formulation of the repricing mechanism based on various forms of the Brownian functionals. These include the excursion time or occupation time below a pre-specified barrier level of the stock price. We manage to obtain analytic representation of the price of repriceable employee stock options. In Section 3, we design effective option valuation algorithms by applying the forward shooting grid approach to obtain the numerical solution of the pricing models. The numerical scheme can be extended to incorporate the feature that allows for potential early exercise of the option due to employment termination and employee's desire for liquidity; and together, the number of repricing can be more than one. In Section 4, we compare the numerical option values obtained from the numerical evaluation of the analytic price formulas and the lattice tree calculations. Such comparison serves to test the validity of the analytic price formulas and numerical accuracy of the lattice tree algorithms. We also examine the impact of various forms of repricing flexibility on the market value of the employee stock options. By examining the sensitivities of the option values with respect to stock price fluctuation and volatility, we deduce the impact of the repricing flexibility on incentive strengths. Concluding remarks are presented in the last section.

2 Model formulation and analytic representation formulas

In this section, we discuss the pricing formulation of an employee stock option whose strike price or/and maturity date may be reset when certain triggering condition is satisfied. Instead of the simple one-touch mechanism where the option terms are reset at the first time that the stock price falls below a prespecified 'barrier' level B, the reset triggering condition is taken to be dependent on the time duration that the stock price stays below B, as measured by various forms of the Brownian functional of the stock price process.

We assume frictionless market, constant interest rate and dividend yield, and that the underlying stock price of the employee stock option observes the following Geometric Brownian process under the risk neutral probability measure Q:

$$\frac{dS_t}{S_t} = (r - q)\, dt + \sigma\, dW_t, \quad S_0 = x, \tag{2.1}$$

where $W_t, t \geq 0$, is a Q-Brownian process and $x > 0, \sigma$ is the constant volatility. It then follows that

$$S_t = x \exp\left(\left(r - q - \frac{\sigma^2}{2}\right) t + \sigma W_t\right). \tag{2.2}$$

For notational convenience, we define

$$\mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma} \quad \text{and} \quad b = \frac{1}{\sigma} \ln \frac{B}{x}, \tag{2.3}$$

where $B < x$ is the pre-specified barrier level for defining the reset triggering condition. Let the date of valuation of the employee stock option be time zero and T be the maturity date of the option. Two common forms of Brownian functional that measures the duration of the stock price staying below the triggering barrier B can be defined as follows.

1. The occupation time $\Gamma_{t,B}$ is the total amount of time that the stock price process is staying below B from time zero up to time t

$$\Gamma_{t,B} = \int_0^t \mathbf{1}_{\{S_u \leq B\}}\, du. \tag{2.4a}$$

   Given α satisfying $0 < \alpha < 1$, the right-continuous inverse of the occupation time is defined by

$$\Gamma^{-1}_{\alpha,B} = \inf\left\{t \geq 0 : \Gamma_{t,B} > \alpha T\right\}, \quad 0 < \alpha < 1. \tag{2.4b}$$

   Here, $\Gamma^{-1}_{\alpha,B}$ is interpreted as the first time at which the total time duration with stock price staying below B reaches α fraction of option's life.

2. The excursion time $H_{t,B}$ at a given time t is the amount of time that the stock price process stays below B in its most recent excursion to the region below B. That is,

$$H_{t,B} = (t - g_{t,B})\mathbf{1}_{\{S_t \leq B\}} \tag{2.5a}$$

   where

$$g_{t,B} = \sup(u \leq t : S_u = B). \tag{2.5b}$$

   Given α satisfying $0 < \alpha < 1$, the right-continuous inverse of the excursion time is defined by

$$H^{-1}_{\alpha,B} = \inf\{t \geq 0 : H_{t,B} > \alpha T\}, \quad 0 < \alpha < 1. \tag{2.6}$$

   Here, $H^{-1}_{\alpha,B}$ is the first time at which the excursion time to the region below B reaches α fraction of the option's life.

The stopping time $\widehat{t}$ at which repricing of the employee stock option is triggered depends on the choice of the Brownian functional, barrier level B and parameter α. For example, suppose we choose the random trigger time $\widehat{t}$ to be governed by $H^{-1}_{\alpha,B}$, then the strike or the maturity date of employee stock option are reset when the excursion time below B reaches α fraction of option's life. In some option contractual design, a black-out period of repricing may be specified. Let δ be the length of the time period prior to the original maturity date within which repricing is not allowed. In this case, repricing can occur only if $\widehat{t} < T - \delta$.

In this paper, we consider the most general case where both strike and maturity are reset and there is a black-out period δ. Upon repricing, let the strike price be reset to K' and maturity date to T'. The terminal option payoff $(S_T - K)^+$ is paid at time T if the triggering condition has not been met prior to $T - \delta$, otherwise the payoff becomes $(S_{T'} - K')^+$ paid at time T' if the triggering time $\widehat{t}$ occurs prior to $T - \delta$. Note that $\widehat{t}$ can be either $\Gamma^{-1}_{\alpha,B}$ or $H^{-1}_{\alpha,B}$, depending on the specification of the Brownian functional used as the criterion of trigger. Suppose the employee stock option is European style, the value of the employee stock option at time zero with repricing flexibility is given by

$$V = E_Q\left[e^{-rT}(S_T - K)^+\mathbf{1}_{\{\widehat{t}\geq T-\delta\}} + e^{-rT'}(S_{T'} - K')^+\mathbf{1}_{\{\widehat{t}< T-\delta\}}\right], \quad (2.7)$$

where E_Q is the expectation under the risk neutral measure Q conditional on the information at time zero.

There are various choices of setting K' and T'. The reset strike may be chosen to depend on the stock price at the repricing date $\widehat{t}$. In the more interesting case of 6 & 1 repricing, the strike price is set at the stock price on 6 months and 1 day after the repricing date $\widehat{t}$. In this case, we have $K' = S_{\widehat{t}+0.5}$. When dealing with maturity extension, the reset maturity T' may become either $\widehat{t}+T$ or $T+5$. The first case implies that the new option is granted with the same life span as that of the original option while the second case refers to a fixed extension period of 5 years beyond the original maturity date.

The European call option value c_{BS} at time 0 with an initial stock price x, strike K and maturity T is given by

$$c_{BS}(x,\tau;K) = E_Q[e^{-rT}(S_T - K)^+], \quad \tau = T. \quad (2.8)$$

For the call price function $c_{BS}(x,\tau;K)$, we follow the notation that the temporal variable in the price function refers to the time to expiry τ of the call option. We would like to decompose the value V at time 0 of the repriceable option into the sum of repricing premium and European call option value so that

$$\begin{aligned} V &= E_Q[e^{-rT'}(S_{T'} - K')^+\mathbf{1}_{\{\widehat{t}< T-\delta\}} \\ &\quad - e^{-rT}(S_T - K)^+\mathbf{1}_{\{\widehat{t}< T-\delta\}}] + c_{BS}(x,T;K). \end{aligned} \quad (2.9)$$

By applying the Girsanov Theorem, we introduce a new probability measure Q_μ such that $Z_t = W_t + \mu t$ is a Q_μ-Brownian process. Under Q_μ, the first term in Eq. (2.9) can be transformed as follows:

$$\begin{aligned} & E_Q[e^{-rT'}(S_{T'} - K')^+ \mathbf{1}_{\{\widehat{t}<T-\delta\}}] \\ = & E_{Q_\mu}\left[e^{-\left(r+\frac{\mu^2}{2}\right)T'} e^{\mu Z_{T'}} \left(xe^{\sigma Z_{T'}} - K'\right)^+ \mathbf{1}_{\{\widehat{t}(Z)<T-\delta\}}\right], \end{aligned} \tag{2.10}$$

where $\widehat{t}(Z)$ is obtained from $\widehat{t}$ by applying appropriate transformation from the stock price process S to the Q_μ-Brownian process Z.

We consider the following various types of reset strike:

1. The reset strike K' is a fixed constant

   By applying the technique of iterated conditional expectation, one can show that

$$\begin{aligned} & E_{Q_\mu}\left[e^{-\left(r+\frac{\mu^2}{2}\right)T'} e^{\mu Z_{T'}} (xe^{\sigma Z_{T'}} - K')^+ \mathbf{1}_{\{\widehat{t}(Z)<T-\delta\}}\right] \\ = & \int_0^{T-\delta}\int_{-\infty}^{\infty} e^{-\left(r+\frac{\mu^2}{2}\right)h} e^{\mu z_h} c_{BS}(xe^{\sigma z_h}, T'-h; K') f(z_h, h)\, dz_h dh, \end{aligned} \tag{2.11}$$

   where $f(z_h, h)$ is the density function of the joint process $(Z_{\widehat{t}(Z)}, \widehat{t}(Z))$. Combining Eqs. (2.9) and (2.11), the analytic representation formula for the value of the employee stock option with repricing flexibility can be expressed as

$$\begin{aligned} V = & c_{BS}(x, T; K) \\ & + \int_0^{T-\delta}\int_{-\infty}^{\infty} e^{-\left(r+\frac{\mu^2}{2}\right)h} e^{\mu z_h} \left[c_{BS}(xe^{\sigma z_h}, T'-h; K')\right. \\ & \left. - c_{BS}(xe^{\sigma z_h}, T-h; K)\right] f(z_h, h)\, dz_h dh \end{aligned} \tag{2.12}$$

2. Resetting the strike price to the prevailing stock price on the trigger date: $K' = S_{\widehat{t}}$

   Using a similar argument as above, we obtain

$$\begin{aligned} V = & c_{BS}(x, T; K) \\ & + \int_0^{T-\delta}\int_{-\infty}^{\infty} e^{-\left(r+\frac{\mu^2}{2}\right)h} [e^{(\mu+\sigma)z_h} c_{BS}(x, T'-h; x) \\ & - e^{\mu z_h} c_{BS}(xe^{\sigma z_h}, T-h; K)] f(z_h, h)\, dz_h dh. \end{aligned} \tag{2.13}$$

3. Resetting the strike price to the prevailing stock price on 6 months and 1 day after the trigger date: $K' = S_{\widehat{t}+0.5}$ (called 6 & 1 repricing)

   Let $c_{fwd}(x, \tau; \Delta)$ denote the price function of a European call option with time to expiry τ conditional on the option that is forward starting Δ period from now with the strike price set at the prevailing stock price on the forward starting date. We then have

$$\begin{aligned} V &= c_{BS}(x, T; K) \\ &+ \int_0^{T-\delta} \int_{-\infty}^{\infty} e^{-\left(r+\frac{\mu^2}{2}\right)h} \left[e^{(\mu+\sigma)z_h} c_{fwd}(x, T' - h; 0.5)\right. \\ &- \left. e^{\mu z_h} c_{BS}(xe^{\sigma z_h}, T - h; K)\right] f(z_h, h)\, dz_h dh. \end{aligned} \qquad (2.14)$$

Joint density function of $(Z_{\widehat{t}(Z)}, \widehat{t}(Z))$
To obtain a closed form analytic representation of the price function of a repriceable employee stock option under various forms of reset strike and maturity, it suffices to find the analytic representation of the density function of the joint process $(Z_{\widehat{t}(Z)}, \widehat{t}(Z))$. When the stopping time $\widehat{t}$ is dependent on either the occupation time $\Gamma_{t,B}(Z)$ or the excursion time $H_{t,B}(Z)$, we are able to derive the joint density of $(Z_{\widehat{t}(Z)}, \widehat{t}(Z))$. The analytic representation formulas may involve an inversion of Laplace transform function. In the literature, computational algorithms for performing numerical inversion of Laplace transforms are well developed. Craddock *et al*. (2000) provide a survey of techniques of numerical inversion of Laplace transforms with applications to derivatives pricing.

We consider the joint density of $(Z_{\widehat{t}(Z)}, \widehat{t}(Z))$ under the following two cases: (i) $\widehat{t}(Z) = \Gamma^{-1}_{\alpha,b}(Z)$, (ii) $\widehat{t}(Z) = H^{-1}_{\alpha,b}(Z)$.

1. Occupation time specification

   The occupation time of the Brownian process $\{Z_t : t \geq 0\}$ with the down-barrier b is defined by

$$\Gamma_{t,b}(Z) = \int_0^t \mathbf{1}_{\{Z_u \leq b\}}\, du \qquad (2.15a)$$

   and its corresponding right-continuous inverse is

$$\Gamma^{-1}_{\alpha,b}(Z) = \inf\{t \geq 0 : \Gamma_{t,b}(Z) > \alpha T\}, \quad 0 < \alpha < 1. \qquad (2.15b)$$

To derive the joint law $P(Z_{\Gamma^{-1}_{\alpha,b}(Z)} \in dz, \Gamma^{-1}_{\alpha,b}(Z) \in dt)$, we start with the case $b = 0$. Applying the known results in Karatzas and Shreve's text (1991) and Huggonnier's paper (1999), we can obtain the following analytic expression of the joint law when $b = 0$.

$$
\begin{aligned}
& m(z,t;\alpha T) \\
= \; & P\left(Z_{\Gamma^{-1}_{\alpha,0}(Z)} \in dz, \Gamma^{-1}_{\alpha,0}(Z) \in dt\right) \\
= \; & \frac{1}{\pi}\left[-\frac{z\alpha T}{t^2\sqrt{\alpha T(t-\alpha T)}}\right]\exp\left(-\frac{z^2}{2\alpha T}\right) \\
& + \sqrt{\frac{2}{\pi}}\left(\frac{1}{t}\right)^{3/2}\left(1-\frac{z^2}{t}\right)\exp\left(-\frac{z^2}{2t}\right)N\left(\frac{z(t-\alpha T)}{\sqrt{\alpha T(t-\alpha T)t}}\right),
\end{aligned}
\tag{2.16}
$$

where $N(x)$ is the normal distribution function. For $b \neq 0$, by using the strong Markov property of the Brownian motion and let

$$T_b = \inf\{t : Z_t = b\}, \tag{2.17}$$

we obtain the density function of the joint law as follows

$$
\begin{aligned}
f(z,t) &= P(Z_{\Gamma^{-1}_{\alpha,b}(Z)} \in dz, \Gamma^{-1}_{\alpha,b}(Z) \in dt) \\
&= P(Z_{\Gamma^{-1}_{\alpha,0}(Z)} + b \in dz, \Gamma^{-1}_{\alpha,0}(Z) + T_b \in dt) \\
&= \int_0^{t-\alpha T} -\frac{b}{\sqrt{2\pi u^3}}\exp\left(-\frac{b^2}{2u}\right) m(z-b, t-u; \alpha T)\, du, \\
&\qquad\qquad z \leq b, t > \alpha T,
\end{aligned}
\tag{2.18a}
$$

and

$$f(z,t) = 0 \quad \text{for} \quad z > b. \tag{2.18b}$$

2. Excursion time specification

   Recall the definition of the right-continuous inverse of the excursion time of the Brownian process $\{Z_t : t \geq 0\}$

$$H^{-1}_{\alpha,b}(Z) = \inf\left\{t : \mathbf{1}_{\{Z_t \leq b\}}(t - g_{t,b}(Z)) > \alpha T\right\} \tag{2.19a}$$

   where

$$g_{t,b}(Z) = \sup\{s : s \leq t, Z_s = b\}. \tag{2.19b}$$

It is important to note that $Z_{H^{-1}_{\alpha,b}(Z)}$ and $H^{-1}_{\alpha,b}(Z)$ are independent (Chesney *et al.*, 1997). Using the marginal density functions derived in Chesney *et al.*'s paper, the joint density function is given by

$$\begin{aligned} & f(z,t) \\ = \; & P(Z_{H^{-}_{\alpha,b}(Z)} \in dz, H^{-1}_{\alpha,b}(Z) \in dt) \\ = \; & P(Z_{H^{-}_{\alpha,b}(Z)} \in dz) P(H^{-1}_{\alpha,b}(Z) \in dt) \\ = \; & \frac{b-z}{\alpha T} \exp\left(-\frac{(z-b)^2}{2\alpha T}\right) \mathbf{1}_{\{z \le b\}} \mathcal{L}_s^{-1}\left(\frac{\exp(b\sqrt{2s})}{\Psi(\sqrt{2s\alpha T})}\right), \end{aligned} \tag{2.20}$$

where $\mathcal{L}_s^{-1}$ denotes the Laplace inversion operator and

$$\Psi(x) = 1 + \sqrt{2\pi}x \exp(x^2/2) N(x). \tag{2.21}$$

3 Construction of lattice tree algorithms

One may perform valuation of the above analytic price formulas through numerical computation of the multiple integrals and use of Laplace inversion algorithm. However, numerical valuation of these repriceable options can be obtained through the lattice tree calculations that are commonly used in option valuation. To cope with the path dependence feature of the Brownian functional in the repriceable option models, it is necessary to augment an auxiliary state vector at each node on the lattice tree that simulates the excursion time or occupation time. This numerical technique is called the forward shooting approach in the literature (Barraquand and Pudet, 1996; Kwok and Lau, 2001).

Forward shooting grid technique

We first discuss how to construct the corresponding forward shooting grid algorithm in pricing employee stock options with various forms of reset of strike price and/or maturity date based on the triggering criteria discussed in Section 2. We also show how to extend the forward shooting grid algorithm to include additional features that allow for potential early exercise and multiple repricing. As in usual lattice tree calculations, we simulate the stock price process S_t by a trinomial lattice tree. Let Δt denote the time step, where $\Delta t = T/M$. Here, M is the total number of time steps in the lattice tree. Let

Δx denote the step width of $x = \ln S$, where $\Delta x = \sigma\sqrt{\Delta t}$. We let $V_{j,k}^m$ denote the numerical option value at the m^{th} time level and j jumps from the initial value of $x = \ln S, m = 0, 1, \cdots, M$, and $j = -m, -m+1, \cdots, 1, \cdots, m$. Here, k represents the positive counting index of the path dependent Brownian functional, which may be the excursion time or occupation time. When the integer counting index k reaches the target cap K_{cap}, where $K_{cap} = \alpha T/\Delta t$, repricing is triggered.

Following the formulation of the forward shooting approach in Kwok-Lau's paper (2001), we construct the discrete grid function $g(k, j)$ that simulates the correlated evolution of the Brownian functional with the stock price process. Let $x_j = \ln S_0 + j\Delta x$, the grid function is defined by

(i) occupation time specification

$$g(k, j) = k + \mathbf{1}_{\{x_j \le \ln B\}} \tag{3.1a}$$

(ii) excursion time specification

$$g(k, j) = (k + 1)\mathbf{1}_{\{x_j \le \ln B\}}. \tag{3.1b}$$

We proceed backward in the lattice tree calculations by specifying the terminal payoff of the option as

$$V_{j,k}^M = \max(e^{x_j} - K, 0), \quad -M \le j \le M, \quad k < K_{cap}. \tag{3.2}$$

To model the black-out period that is δ-period prior to expiration during which repricing is not allowed, we define

$$\widetilde{M} = M - \frac{T - \delta}{\Delta t}. \tag{3.3}$$

We follow the usual trinomial calculations for $\widetilde{M} \le m \le M - 1$, where

$$V_{j,k}^m = e^{-r\Delta t}\left(p_u V_{j+1,g(k,j+1)}^{m+1} + p_o V_{j,g(k,j)}^{m+1} + p_d V_{j-1,g(k,j-1)}^{m+1}\right). \tag{3.4}$$

The probability values in the above trinomial scheme are given by

$$p_u = \frac{\nu + c}{2}, \quad p_o = 1 - \nu \quad p_d = \frac{\nu - c}{2}, \tag{3.5a}$$

where

$$\nu = \frac{\sigma^2 \Delta t}{\Delta x} \quad \text{and} \quad c = \left(r - q - \frac{\sigma^2}{2} \right) \frac{\Delta t}{\Delta x}. \qquad (3.5b)$$

When $0 \leq m < \widetilde{M}$, repricing will be triggered when the value of the grid function equals or exceeds K_{cap}. Hence, when $k < K_{cap}$, the lattice tree calculations follow the usual trinomial scheme as depicted in Eq. (3.4). However, when $k = K_{cap}$, we set the nodal option value to be

$$V^m_{j,K_{cap}} = \begin{cases} c_{BS}(\ln x_j, \tau; K') & \text{pre-set new strike equals } K' \\ c_{BS}(\ln x_j, \tau; \ln x_j) & \text{new strike equals prevailing stock price} \\ c_{fwd}(\ln x_j, \tau; 0.5) & \text{6 \& 1 repricing} \end{cases}, \qquad (3.6)$$

according to various policies of strike reset. Here, the time to expiry τ of the new option can be $T - m\Delta t$ if there is no maturity extension or equals T if the new option is granted with the same life span T as that of the original option.

Early exercise feature and multiple repricing

The forward shooting grid approach has the flexibility to incorporate additional features, like the potential early exercise of the option and occurrence of multiple repricing. For the early exercise (or forfeiture) feature, we follow the Carr and Linetsky's intensity approach (2000) of modeling the arrival of the early exercise event as an exogenous point process. Let h_t denote the early exercise intensity with dependence on the stock price S_t and time t. We take

$$h_t = \lambda_f + \lambda_e \mathbf{1}_{\{S_t > K\}}, \qquad (3.7)$$

where λ_f and λ_e are positive constants. Here, λ_f is the constant intensity of early exercise due to the exogenous employment termination (taken to be independent of the stock price) and $\lambda_e \mathbf{1}_{\{S_t > K\}}$ is the constant intensity of the early exercise due to the employee's desire for liquidity (occurring only when the option is in-the-money). To incorporate the above criterion of early exercise, the forward shooting grid algorithm is modified as follows:

$$\begin{aligned} V^m_{j,k} &= \exp\left(-\left(r + \lambda_f + \lambda_e \mathbf{1}_{\{x_j > \ln K\}}\right)\Delta t\right) \\ &\quad \left[p_u V^m_{j+1,g(k,j+1)} + p_o V^m_{j,g(k,j)} + p_d V^m_{j-1,g(k,j-1)}\right] \\ &\quad + (\lambda_f + \lambda_e)\Delta t (e^{x_j} - K)^+. \end{aligned} \qquad (3.8)$$

As deduced from the governing differential equation of the option value function [see Eq. (5) in Carr-Linetsky's paper], the intensity h_t enters into the discount factor term where the discount rate "apparently" increases from r to $r + \lambda_f + \lambda_e \mathbf{1}_{\{x_j > \ln K\}}$. The last term $(\lambda_f + \lambda_e)\Delta t (e^{x_j} - K)^+$ represents the outcome of the early exercise payoff $(e^{x_j} - K)^+$ that occurs with probability $(\lambda_f + \lambda_e)\Delta t$ over the time interval Δt when the option is in-the-money (that is, $e^{x_j} - K > 0$).

To allow for multiple repricing, it is necessary to modify the auxiliary condition at $k = K_{cap}$. Instead of setting $V^m_{j,K_{cap}}$ to be the Black-Scholes price function [as shown in Eq. (3.6)], we simply set $V^m_{j,K_{cap}}$ to be the price function of the repriceable option with one repricing right less. The lattice tree calculations continue with nesting iterations on the count of repricing right, with the count decreasing by one whenever a repricing occurs (the occurrence is contingent upon the satisfaction of the repricing trigger criterion).

4 Numerical calculations and examination of pricing behaviors

We performed numerical calculations of the value of the employee stock option under various reset policies on the strike and maturity and different repricing trigger mechanisms. Unless otherwise stated, the parameter values of the option model are

$$S_0 = 100, K = 100, B = 90, \sigma = 20\%, r = 5\%, q = 2\%, T = 5 \text{ and } \alpha = 10\%.$$

For the reset policy on the strike price, we consider

(i) 6 & 1 synthetic repricing: $K' = S_{\widehat{t}+0.5}$;

(ii) new strike set at the prevailing stock price at trigger moment: $K' = S_{\widehat{t}}$;

(iii) new strike set at the barrier level: $K' = B$.

Let T' be the new maturity date of the new option. For the reset policy on the maturity date, we allow

(i) no maturity extension at all: $T' = T$;

(ii) new maturity date set at 5 years from the trigger date: $T' = \widehat{t} + 5$.

In Tables 1 and 2, we list the numerical option values of the employee stock option obtained from numerical valuation of the analytic price formulas and lattice tree calculations using the forward shooting gird algorithm under various reset policies and repricing trigger mechanisms (occupation time criteria and excursion time criteria in Table 1 and Table 2, respectively). We also list the percentage gain in option value of the repriceable options with reference to the option value of the vanilla counterpart (considered as the premium of the repriceable feature as percentage of the vanilla option value). As repricing flexibility always leads to an increase in option value, so we always have a positive value of premium for all types of reset criteria. The numerical option values are obtained from the lattice tree calculations using varying number of time steps N. Taking the numerical values from valuation of analytic formulas as "exact", the percentage errors of the numerical results from lattice tree calculations are seen to be small, typically less than 0.2%. The good agreement between the two sets of numerical results serves to verify the validity of the analytic formulas.

In Table 3, we list the numerical values of the executive stock options which allow for double repricing flexibility and possibility of early termination. The numerical calculations are based on the lattice tree algorithm defined in Eq. (3.8), together with the use of the early exercise intensity defined in Eq. (3.7). We use the occupation time specification as the repricing trigger mechanism and the strike reset and maturity reset criteria are defined by $K' = S_{\widehat{t}}$ and $T' = \widehat{t} + 5$, respectively. The value of the executive stock option that allows for double repricing (shown in Table 3 with $\lambda_f = \lambda_e = 0$) is seen to be higher than that of the counterpart with single repricing (shown in Tables 1 and 2 with $K' = S_{\widehat{t}}$ and $T' = \widehat{t} + 5$). Obviously, option value decrease with higher propensity of early exercise since this leads to potential shorter life of the option. Hence, as λ_f and/or λ_e increases, the option value decreases. Comparing the two intensity parameters, λ_f has a stronger influence on the drop of the option value.

Impact of repricing on option values and employees' incentives

First, we would like to comment on the impact of reset criteria and various mechanisms of repricing trigger on the value of an employee stock option. We observe from Table 1 that the strike reset policy which sets the new option strike price at the prevailing stock price gives the highest option value compared to other strike reset policies. This is quite obvious as the prevailing stock price is expected to be lower than the barrier level or the stock price at 6 months later, and a lower new strike price means a higher value for the new call option. Comparing the two sets of option values in Tables 1 and 2, the option values corresponding to the triggering criterion that is based on excursion time specification (entries in Table 2) are lower than those under the occupation time specification (entries in Table 1). This is intuitive since repricing is more difficult to be triggered under the excursion time specification. We also plot the value of repriceable option against various parameters in the pricing model, like stock price (see Figure 1), option maturity (see Figure 2), stock price volatility (see Figure 3), repricing barrier level (see Figure 4). The option values are seen to be increasing with respect to each of these parameter values. In Figure 2, we observe that the long-maturity options are less sensitive to the duration of occupation time or excursion time specified for triggering repricing.

Next, we would like to compare the incentive effects of the vanilla (non-repriceable) executive stock options with their repriceable counterparts. The natural question: would the repricing feature enhance or decrease the incentives for the executives to increase the stock price and stock price volatility? Though the measurement of the exact incentives for a particular executive would depend on the executive's personal risk preference and wealth level, we use the risk neutral option values and their comparative statics as the proxies for the assessment of incentives of increasing stock price / stock return volatility. In Figure 5, we show the plot of the option delta (derivative of the option value with respect to the stock price) against stock price of the repriceable executive stock options under the occupation time repricing mechanism and with varying forms of strike reset and maturity reset. We observe that under all repricing criteria, the deltas of the repriceable options are always lower than those of the vanilla counterparts. This is consistent with the usual intuition that the repricing flexibility leads to a lower incentive for the employees to increase the stock price, thus a lower option delta value. The delta values are seen to differ quite significantly with respect to different reset policies and stock price level. The option delta values increase with higher stock price level, a typical pricing property of option type derivatives. The deltas assume lower values when the strike reset criterion is based on the prevailing stock price or stock price in 6 months later. Another comparative static is the option vega, which is the derivative of the option price with respect to the volatility σ of the stock price. Higher option vega of the option means a higher incentive for the option holder to increase the volatility (firm risk). In Figure 6, we plot the option vega against volatility σ under varying reset policies on the strike price and maturity. For all cases of repricing, the vega values of the executive stock options with repricing flexibility are always higher than those of the vanilla option. Highest vega values occur for the case when the strike price is reset to the prevailing stock price and the new maturity is extended to 5 years after the trigger date.

5 Conclusion

Retention enhancing features in compensation package are more important when there is a higher perceived retention risk and better outside employment opportunities for the firm's employees. Empirical studies on employee stock options have shown that repricing benefits shareholders since it enhances retention of key managers and employees in the presence of underwater options. In this paper, we propose pricing formulations of repriceable employee stock options in which we model various forms of repricing mechanisms that are based on the occupation time and excursion time of the stock price process. Our pricing models allow for different types of strike price reset and maturity extension upon the trigger of repricing. We manage to obtain analytic representation of the price function of the repriceable options in terms of a multiple integral that involves the density function of the joint process of stock price and trigger time of repricing. The option values can also be computed effectively using trinomial tree algorithms embedded with the forward shooting grid technique, by appending a grid function at each lattice node to capture the correlated evolution of the stock price process and its Brownian functional. The trinomial tree algorithm is also extended to price repriceable executive stock options that allow for multiple repricing flexibility and early termination. Impact of various repricing mechanisms and reset policies on the employee option values and comparative statics have also been studied. Depending on the chosen criteria on strike reset and maturity reset, we observe that the option delta (option vega) of the repriceable stock options are always lower (higher) than those of the non-repricable counterparts. These observations indicate that the various reset polices have negative impact on employees' incentives to increase the stock price and positive impact on increasing the stock return volatility. Our reported results on incentive effects may shed insight on the design of optimal executive stock option plans that better align the interests of the employees with those of their employers.

References

[1] Acharya, V.V., John, K., and Sundaram, R.K. (2000). On the optimality of resetting executive stock options. Journal of Financial Economics, 57, 65-101.

[2] Barraquand, J., and Pudet, T. (1996). Pricing of American path-dependent contingent claims, Mathematical Finance, 6(1), 17-51.

[3] Brenner, M., Sundaram, R.K., and Yermack, D. (2000). Altering the terms of executive stock options. Journal of Financial Economics, 57, 103-128.

[4] Carr, P., and Linetsky, V. (2000). The valuation of executive stock options in an intensity-based framework. European Finance Review, 4, 211-230.

[5] Carter, M.E., and Lynch, L.J. (2003). The effect of stock option repricing on employee turnover. Working paper of University of Pennsylvania and University of Virginia.

[6] Chance, D.M., Kumar, R. and Todd, R.B. (2000). The 'repricing' of executive stock options. Journal of Financial Economics, 57, 129-154.

[7] Chen, M. (2004). Executive option repricing, incentives, and retention. Journal of Finance, 59(3), 1167-1199.

[8] Chesney, M., Cornwall, J., Jeanblanc-Picqué M., Kentwell, G. and Yor, M. (1997). Parisian pricing. Risk, 10(1), 77-79.

[9] Chidambaran, N.K., and Prabhala, N.R. (2003). Executive stock option repricing, internal governance mechanisms and management turnover. Journal of Financial Economics, 69(1), 153-189.

[10] Corrado, C., Jordan, B.D., Miller, T.W. Jr., and Stansfield, J.J. (2001). Repricing and employee stock option valuation. Journal of Banking and Finance, 25, 1059-1082.

[11] Craddock, M., Heath, D. and Platen, E. (2000). Numerical inversion of Laplace transforms: a survey of techniques with applications to derivative pricing. Journal of Computational Finance, 4(1), 57-81.

[12] Ferri, F. (2004). Structure of option repricings: Determinants and consequences. Working paper of New York University.

[13] Huggonnier, J. (1999). The Feynman-Kac formula and pricing occupation time derivatives. International Journal of Theoretical and Applied Finance, 2(2), 153-178.

[14] Johnson, S.A., and Tian, Y.S. (2000). The value and incentive effects of nontraditional executive stock option plans. Journal of Financial Economics, 57, 3-34.

[15] Karatzas. I. and Shreve, S. (1991). *Brownian motion and stochastic calculus*, 2nd edition, Springer-Verlag, New York.

[16] Lau, K.W. and Kwok, Y.K. (2001). Pricing algorithms for options with exotic path-dependence. Journal of Derivatives, Fall issue, 1-11.

[17] Zheng, L. (2004). Six-month-one-day employee stock option repricing: an examination of accounting considerations and incentive implications. Working paper of the University of Southern California.

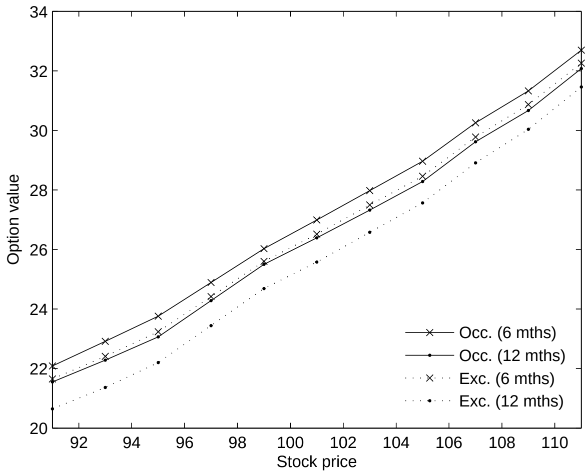

Figure 1 Plot of employee stock option value against stock price under varying repricing criteria, where the occupation time and excursion time can be 6 months or 12 months.

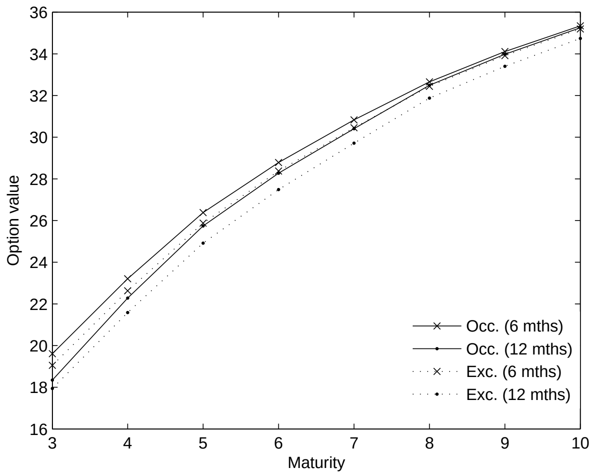

Figure 2 Plot of employee stock option value against maturity under varying repricing criteria, where the occupation time and excursion time can be 6 months or 12 months.

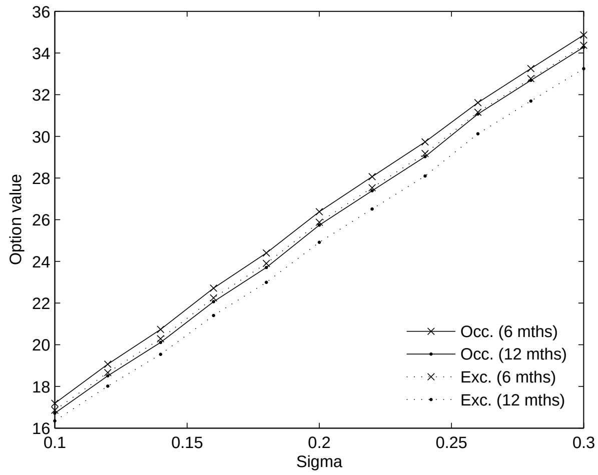

Figure 3 Plot of employee stock option value against stock price volatility σ under varying repricing criteria, where the occupation time and excursion time can be 6 months or 12 months.

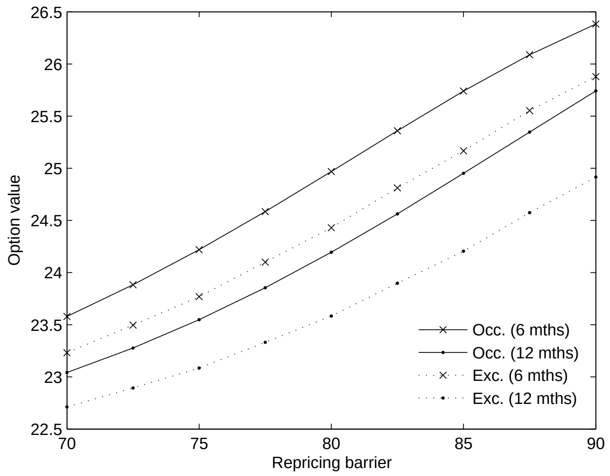

Figure 4 Plot of value of employee stock option against repricing barrier under varying repricing criteria, where the occupation time and excursion time can be 6 months or 12 months.

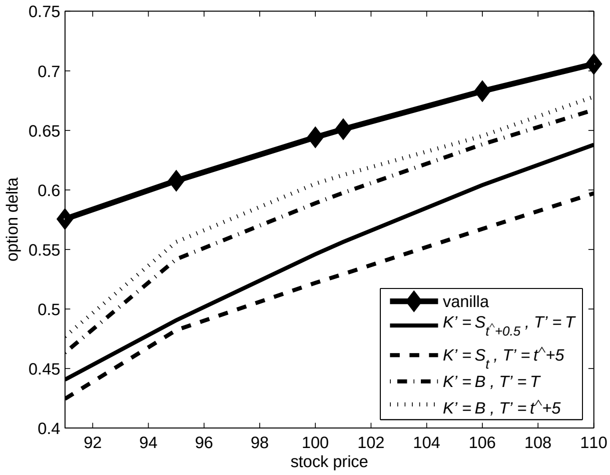

Figure 5 Plot of option delta against stock price under varying strike reset and maturity reset criteria.

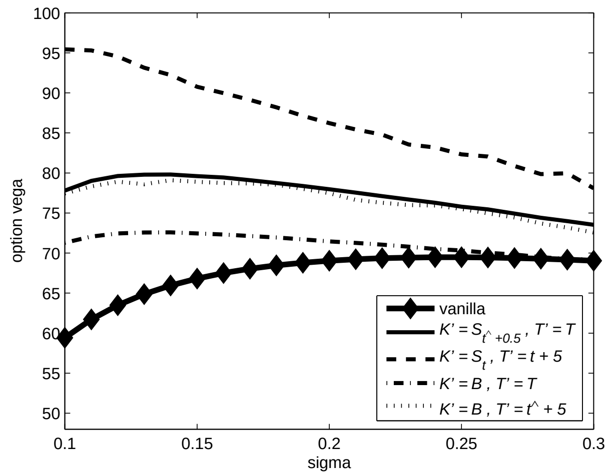

Figure 6 Plot of option vega against stock price volatility under varying strike reset and maturity reset criteria.

		analytic formula		lattice tree calculation		
		option value	percentage gain	$N = 72$	$N = 288$	$N = 512$
$K' = S_{\widehat{t}+0.5}$	$T' = T$	24.61	11.81%	24.62	24.60	24.60
	$T' = \widehat{t} + 5$	26.54	20.58%	26.53	26.58	26.60
$K' = S_{\widehat{t}}$	$T' = T$	25.28	14.83%	25.27	25.26	25.27
	$T' = \widehat{t} + 5$	27.06	22.94%	27.03	2.708	27.11
$K' = B$	$T' = T$	23.49	6.04%	23.26	23.31	23.35
	$T' = \widehat{t} + 5$	25.30	13.67%	25.04	25.16	25.22

Table 1 Comparison of the numerical option values of executive stock options obtained from valuation of analytic price formulas and lattice tree calculations (with varying number of time steps N). The repricing trigger mechanism is based on the specification of occupation time.

		analytic formula		lattice tree calculatons		
		option value	percentage gain	$N = 72$	$N = 288$	$N = 512$
$K' = S_{\widehat{t}+0.5}$	$T' = T$	24.27	10.26%	24.47	24.42	24.42
	$T' = \widehat{t} + 5$	25.83	17.35%	26.23	26.16	26.16
$K' = S_{\widehat{t}}$	$T' = T$	25.08	12.67%	25.04	24.97	24.97
	$T' = \widehat{t} + 5$	26.67	19.26%	26.66	26.57	26.57
$K' = B$	$T' = T$	23.04	4.22%	23.02	22.97	22.97
	$T' = \widehat{t} + 5$	24.62	11.85%	24.64	24.57	24.57

Table 2 Comparison of the numerical option values of executive stock options obtained from valuation of analytic price formulas and lattice tree calculations (with varying number of time steps N). The repricing trigger mechanism is based on the specification of excursion time.

	occupation time (double repricing)		excursion time (double repricing)	
	option value	percentage gain	option value	percentage gain
$\lambda_f = 0, \lambda_e = 0$	28.18	28.02%	27.23	23.73%
$\lambda_f = 0, \lambda_e = 0.05$	26.80	27.77%	25.95	17.89%
$\lambda_f = 0, \lambda_e = 0.1$	25.58	16.20%	24.80	12.66%
$\lambda_f = 0.05, \lambda_e = 0$	25.49	15.78%	24.64	11.95%
$\lambda_f = 0.05, \lambda_e = 0.05$	24.33	10.55%	23.56	7.05%
$\lambda_f = 0.05, \lambda_e = 0.1$	23.30	5.86%	22.59	2.64%
$\lambda_f = 0.1, \lambda_e = 0$	23.20	5.40%	22.45	1.97%
$\lambda_f = 0.1, \lambda_e = 0.05$	22.23	0.99%	21.53	−2.17%
$\lambda_f = 0.1, \lambda_e = 0.1$	21.36	−2.98%	20.71	−5.91%

Table 3 Values of the executive stock options with allowance of double repricing and possibility of early termination [as characterized by the intensity parameters λ_e and λ_f defined in Eq. (3.7)].