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# Effects of Callable Feature on Early Exercise Policy

YUE KUEN KWOK* maykwok@ust.hk
*Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong*

LIXIN WU malwu@ust.hk
*Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong*

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**Abstract.** Convertible bonds and American warrants commonly contain the provision of the callable feature which allows the issuer to buy back the derivative at a predetermined recall price. Upon recall, by virtue of the early exercise privilege embedded in an American style derivative, the holder may choose either to exercise his derivative or to sell it back to the issuer. Normally, there is a notice period requirement on the recall, that is, the decision of the holder to exercise or to receive the cash is made at the end of the notice period. Also, the period of recall provision may cover only part of option's life. In this article, we examine the effect of the callable feature (with the notice period requirement) on the early exercise policy of a callable American call option. The optimal calling policy for the issuer is explored where the value of the American option is minimized among all possible recall policies. Without the notice period requirement, the critical asset price boundary of the callable American call is identical to that of the American capped call. When the notice period requirement is imposed, the critical asset price (considered as a function of time to expiry $\tau$) first increases with $\tau$, reaches some maximum value, then decreases with $\tau$. Several approaches of designing numerical algorithms for the valuation of the callable American option are also presented.

**Keywords:** American call, callable feature, optimal recall policies, notice period.

## 1. Introduction

The callable (redeemable) feature embedded in a financial derivative gives the right to the issuing company to buy back the derivative from the holder at a predetermined recall price. It is quite common for convertible bonds to be issued with the provision of the callable feature. Warrants with the combination of early exercise feature and callable feature are also available in the financial markets. Two examples are quoted here: Three Year Redeemable Common Stock Purchase Warrants of Ontro, Inc. and Redeemable Warrants of Frontline Communications Corp. More examples of callable warrants can be found in the paper by Burney and Moore (1997).

For a callable American warrant, in the event of a recall by the issuer, the holder is given a *notice period*, usually 30 days, to decide whether to exercise his warrant or to sell it back to the issuer at the recall price. The following terms of the callable (redeemable) feature are typical:

> "The Warrants are redeemable by the Company at any time commencing (some date), upon notice of not less than 30 days, at a price of (certain dollars) per Warrant, provided that the closing bid quotation of the Common Stock on all 20 trading days

* Corresponding author

ending on the third day prior to the day on which the Company gives notice has been at least 150% of the then effective exercise price of the Warrants."

The pricing model of an American option with the callable feature was first discussed in the seminal paper by Merton (1973). Merton claimed that there is no closed form analytic formula available for the price of a callable American call option, and only the special case of the perpetual callable American call option was fully analyzed. For other convertible securities, the optimal calling policies for convertible bonds have been examined by Ingersoll (1977a, 1977b) and Brennan and Schwartz (1977). A more recent article (Burney and Moore, 1997) considered the valuation model for callable warrants. However, their analysis does not allow for early voluntary exercise by the holder.

It has been observed empirically in the financial markets that issuers may not recall a convertible security even when its conversion value exceeds the recall price. Some research papers reported that this "delayed recall" phenomena on convertible debts may be attributed to the presence of the notice period requirement and tax incentive (Asquith, 1995; Asquith and Mullins, 1991; Jaffee and Shleifer, 1990). Contrarily, Ingersoll (1977b, p. 469) observed that the introduction of the notice period requirement into the perpetual convertible debt model reduces the critical firm value at which the issuer should recall the debt optimally.

In this paper, using the callable American call option as an example, we would like to examine the connection between the callable feature and early exercise policy of a callable American style derivative. What should be the optimal calling policy adopted by the issuer so that the value of the American callable call option is minimized among all possible recall policies? In our analysis, we assume the usual Black-Scholes framework where the underlying asset price follows the lognormal diffusion process and the riskless interest rate is constant. For simplicity of analysis, the recall price is taken to be constant. In particular, we take into consideration the effect of the notice period requirement on the critical asset price at which the option should be recalled optimally by the issuer. To be more general, we also consider partial recall period where the right to recall is limited to only part of the option's life.

The paper is organized as follows. In Section 2, the formulation of the pricing model for the callable American call option is discussed. The characterization of the callable feature on the optimal recall policy and early exercise policy is analyzed. In several cases, we manage to obtain the analytical price formulas for the callable American call option model. The similarities and differences between the callable American call option and the American capped call option (Broadie and Detemple, 1995) are also addressed. In Section 3, three numerical algorithms for the valuation of the callable American call option are constructed. Interestingly, though these numerical algorithms are derived based on different financial arguments, it turns out that they give identical numerical results. Numerical results for the critical boundaries and option values for the callable American call option models under a variety of callable conditions, like the presence of notice period requirement and partial period of recall, are presented in Section 4. The paper ends with summaries and concluding remarks in the last section.

## 2. Characterization of the Optimal Calling Policies

It is relatively straightforward to write down the governing equation for the pricing model of an American call option with the callable feature. Let $K$ be the recall price (assumed constant) paid to the holder of the call option when it is recalled by the issuer. With the usual Black-Scholes assumptions, the call option value $C(S,\tau)$ satisfies the Black-Scholes equation

$$\frac{\partial C}{\partial \tau} = \frac{\sigma^2}{2} S^2 \frac{\partial^2 C}{\partial S^2} + (r-q) S \frac{\partial C}{\partial S} - rC, \quad 0 < S < S^*(\tau), \quad \tau > 0, \tag{1}$$

where $r$ and $q$ are the constant riskless interest rate and dividend yield respectively, $\sigma$ is the constant volatility, $\tau$ is the time to expiry, $S$ is the asset price and $S^*(\tau)$ is the critical level of the asset price at which the option ceases to exist. The premature termination of the option occurs either when the option is exercised at the will of the holder or recalled at the discretion of the issuer. To simplify the analysis of the optimal early exercise policy and recall policy, the model assumes continuous dividend yield of the underlying asset rather than discrete disbursements of dividends. The terminal payoff function and boundary condition at $S = 0$ are given by

$$C(S,0) = \max(S - X, 0) \quad \text{and} \quad C(0,\tau) = 0, \tag{2}$$

respectively, where $X$ is the strike price of the option.

First, suppose there is *no notice period requirement*, we argue that the value of the callable American call when it stays alive must be less than or equal to the recall price $K$. By following similar argument used in deducing the optimal recall policy of a convertible bond (Brennan and Schwartz, 1977), the American call will be recalled as soon as its value if recalled is equal to the value if not recalled. In other words, the option can stay unrecalled only if its value stays below its recall price (though the American option may be voluntarily exercised prior to expiry even its value is below $K$). We represent this constraint condition on the call value by

$$C(S,\tau) \le K, \quad \text{for } 0 \le S \le S^*(\tau), \quad \tau \ge 0. \tag{3}$$

The above constraint dictates that the critical asset price $S^*(\tau)$ cannot go beyond $K + X$. If otherwise, the corresponding intrinsic value of the American call option at asset price $S$ satisfying $S^*(\tau) > S > K + X$ would be higher than $K$.

The boundary conditions along $S^*(\tau)$ depend on whether the option is exercised voluntarily by the holder or recalled optimally by the issuer. It is seen that the boundary value $C(S^*,\tau)$ takes the form:

(i) when the call option is recalled optimally by the issuer

$$C(S^*,\tau) = K \tag{4a}$$

(ii) when the call option is exercised at the will of the holder

$$C(S^*,\tau) = S^* - X \quad \text{and} \quad \frac{\partial C}{\partial S}(S^*,\tau) = 1. \tag{4b}$$

Apparently, the payoff upon recall is equal to $\max(S^* - X, K)$ since the holder can either exercise the option or receive cash amount $K$. However, since $S^* \leq K + X$, we have $\max(S^* - X, K) = K$ and thus leads to Eq. (4a).

*Partial Recall Period*

It may occur that the recall privilege is granted to the holder only for part of the option's life. The constraint condition on the option value as given in Eq. (3) is imposed only during the time period when the recall privilege is activated. Across the time instant at which the recall provision is either cancelled or activated, it is expected that the option value remains continuous but the critical asset price $S^*(\tau)$ may exhibit jump.

*Presence of Notice Period Requirement*

When there is notice period requirement, upon recall by the issuer, the holder has the right to delay his decision to exercise or to receive the cash until the end of the notice period. This right can be regarded as a European option. The life of this vested option equals the length of the notice period, $t_n$. Let $c_n(S, t_n)$ denote the value of the vested option, the terminal payoff of which is given by

$$c_n(S, 0) = \max(S - X, K) = K + \max(S - K - X, 0). \tag{5}$$

Hence, $c_n(S, t_n)$ equals the sum of $Ke^{-r\tau}$ and the value of a European call option on the same underlying asset and with strike price $K + X$. The constraint condition (see Eq. (3)) on the call value has to be modified as

$$C(S, \tau) \leq c_n(S, t_n), \quad \text{for } 0 \leq S \leq S^*(\tau), \quad \tau \geq 0. \tag{6}$$

With the notice period requirement, the boundary condition along $S^*(\tau)$ as stated in Eq. (4a) has to be modified to become

$$C(S^*, \tau) = c_n(S^*, t_n). \tag{7}$$

In the following subsections, we analyze the optimal calling policies under the simplified situation, namely, recall provision is activated throughout the whole life of the option and there is no notice period requirement. We manage to deduce the behaviors of the critical asset price boundary $S^*(\tau)$ and derive the corresponding closed form analytical price formulas. The resemblance between the callable American call option under these simplified assumptions and the American capped call option is examined. The pricing behaviors of the perpetual American callable call option is also discussed.

The optimal calling policies are complicated by the presence of the notice period requirement and partial recall period. The discussion of the quantitative behaviors of $S^*(\tau)$ and option value under the conditions of partial recall period and finite notice period requirement will be discussed in Section 4.

### 2.1. Behaviors of the Critical Asset Price, $S^*(\tau)$

Imagine that when the recall price $K$ is sufficiently high, there is no incentive for the issuer to recall the American call option so that the recall privilege is rendered superfluous. The callable American call simply resembles the usual non-callable counterpart. On the contrary, when $K$ is sufficiently low, one may envision that forced conversion of the callable option may occur prior to voluntary early exercise. The justification of the above conjectural statements will be made subsequently.

There is a close relation between $S^*(\tau)$ and $\widetilde{S}^*(\tau)$, where $\widetilde{S}^*(\tau)$ denotes the critical asset price at which the corresponding non-callable counterpart should be exercised optimally. Recall that given a set of parameter values $X$, $\sigma$, $r$ and $q$, $\widetilde{S}^*(\tau)$ can be uniquely determined. Also, $\widetilde{S}^*(\tau)$ is known to be a monotonically increasing function of $\tau$ with its lower bound (Kim, 1990; Carr *et al.*, 1992)

$$\widetilde{S}^*(0^+) = \max\left(X, \frac{r}{q}X\right). \tag{8}$$

Consider the solution to the equation

$$\widetilde{S}^*(\tau) - X = K, \tag{9}$$

with $\tau$ as the unknown variable. The interval for $\tau$ is $[0, T]$, where $T$ is the life of the callable American call. Since $\widetilde{S}^*(\tau)$ is monotonically increasing in $\tau$, there is no solution if $K < \widetilde{S}^*(0^+) - X$ or $K > \widetilde{S}^*(T) - X$; and a unique solution $\tau^*$ exists when $K \in [\widetilde{S}(0^+) - X, \widetilde{S}^*(T) - X]$.

(1) $K > \widetilde{S}^*(T) - X$
When $S < \widetilde{S}^*(\tau)$, the value of the unrecalled callable American call should be less than $\widetilde{S}^*(\tau) - X$ since the call value is an increasing function of $S$ and the call value becomes $\widetilde{S}^*(\tau) - X$ at $S = \widetilde{S}^*(\tau)$. The unrecalled option value is seen to be less than or equal to $\widetilde{S}^*(T) - X$ (since $S^*(\tau) \le S^*(T)$ for $\tau \le T$), and in turns less than $K$. In summary, we have $C(S, \tau) < \widetilde{S}^*(\tau) - X \le \widetilde{S}^*(T) - X < K$. The recall by the issuer would raise the call value to become $K$; and obviously, this is not an optimal recall policy. Consequently, the issuer would not recall the call option throughout the remaining life of the option since the above argument holds for all $\tau \in [0, T]$. When the recall privilege is forfeited, the callable American call simply resembles its non-callable counterpart. In this case, the American call would be optimally exercised by the holder at $\widetilde{S}^*(\tau)$, so

$$S^*(\tau) = \widetilde{S}^*(\tau), \quad \text{for all } \tau \in [0, T]. \tag{10a}$$

(2) $\widetilde{S}(0^+) - X \le K \le \widetilde{S}^*(T) - X$
Let $\tau^*$ denote the unique solution to Eq. (9) under the given stated condition on $K$. We consider two separate cases, namely, $\tau < \tau^*$ and $\tau \ge \tau^*$.

(a) When $\tau < \tau^*$, we have $K > \widetilde{S}^*(\tau) - X$ since $\widetilde{S}^*(\tau)$ is a monotonically increasing function of $\tau$. The argument in part (1) remains valid, that is, it is not optimal for the issuer to recall the American call. Within this period, the callable American call reduces to its non-callable counterpart, and so $S^*(\tau) = \widetilde{S}^*(\tau)$ for $\tau < \tau^*$.

(b) When $\tau > \tau^*$, we have $\widetilde{S}^*(\tau) > K + X$. On the other hand, it has been argued earlier that $S^*(\tau) \leq K + X$. Therefore, it is *impossible* to have $S^*(\tau) = \widetilde{S}^*(\tau)$ for $\tau > \tau^*$. First, we show that the option should not be early exercised when $S < K + X$ at $\tau > \tau^*$. To explain why this is so, we observe that the call option value at $\tau = \tau^*$ is above the intrinsic value for all values of $S$ satisfying $S < K + X$. As a longer lived American option should be more expensive, it is not optimal for the holder to exercise when $S < K + X$ at $\tau > \tau^*$ since the American option value drops to the intrinsic value upon early exercise. Should the issuer utilize the recall privilege at $S < K + X$? Since the call value equals $K$ upon recall, the call value can be lowered by choosing to recall at a higher critical asset value, provided that the critical asset value does not shoot higher than $K + X$. The optimal policy is then to recall at $S = K + X$.

(c) When $\tau = \tau^*$, it is indifferent to the holder whether the option is early exercised or recalled at $S^*(\tau) = K + X$.

In summary, we then conclude that

$$S^*(\tau) = \begin{cases} \widetilde{S}^*(\tau), & \tau < \tau^* \\ K + X, & \tau \geq \tau^* \end{cases}$$
$$= \min(\widetilde{S}^*(\tau), K + X). \tag{10b}$$

Note that $S^*(\tau)$ is continuous at $\tau = \tau^*$.

(3) $K < \widetilde{S}(0^+) - X$

Now, $K < \widetilde{S}(\tau) - X$, for all $\tau \in [0, T]$, since $S^*(\tau)$ is an increasing function of $\tau$. Therefore, the argument in part (2b) corresponding to $\tau > \tau^*$ is applicable, that is, the option will be recalled at $K + X$ and there will be no voluntary early exercise prior to recall. This is the optimal strategy adopted for the whole life of the option. We then have

$$S^*(\tau) = K + X, \quad \tau \in [0, T]. \tag{10c}$$

*Distribution Free Results*

In the above discussion on the critical asset price $S^*(\tau)$, we pay no regard to the lognormal distribution assumption of the underlying asset price movement, and so the conclusions on $S^*(\tau)$ can be applied to other distribution models as well. However, the analytical price formulas derived in Subsection 2.2 do depend on the lognormal distribution assumption of $S$.

### 2.2. *Analytical Price Formulas*

Since the optimal recall policy and exercise policy are relatively simple under the assumptions of no notice period and full period of recall provision, it is quite straightforward to derive the analytical price formulas under the following scenarios on $K$.

(1) $K > \widetilde{S}^*(T) - X$
Since the recall privilege is rendered superfluous, the value of the callable American call option is exactly the same as that of its non-callable counterpart.

(2) $\widetilde{S}^*(0^+) - X \le K \le \widetilde{S}^*(T) - X$
The callable American call option resembles a compound option, where it behaves like the usual non-callable American call when $\tau < \tau^*$ and it becomes a European barrier call option when $\tau \ge \tau^*$, with constant up-and-out barrier $K + X$ and rebate payment $K$ when the barrier is breached. Let $\psi_B(S_{\tau^*}; S)$ denote the risk neutral transition density function of the asset price $S_{\tau^*}$ at $\tau^*$, given $S$ at time to expiry $\tau$ (which is greater than $\tau^*$), on condition that the asset price never breaches the up-and-out barrier: $S = K + X$ during the time period. Also, let $Q(\omega; S)$ be the density function of the first passage time for the asset price to touch the up-barrier in the time interval from $\tau$ to $\tau^*$. The value of the callable American call is given by

$$C(S,\tau) = e^{-r(\tau-\tau^*)} \int_0^{K+X} \widetilde{C}(S_{\tau^*},\tau^*)\psi_B(S_{\tau^*}; S)\, dS_{\tau^*} + K\int_{\tau^*}^{\tau} e^{-r\omega} Q(\omega; S)\, d\omega,$$
$$0 < S < K + X,\ \tau^* < \tau \le T, \tag{11}$$

where $\widetilde{C}(S_{\tau^*}, \tau^*)$ is the value of the corresponding non-callable American call at time to expiry $\tau^*$,

$$\psi_B(S_{\tau^*}; S) = \frac{1}{S_{\tau^*}\sqrt{2\pi\sigma^2(\tau-\tau^*)}}\left[\exp\left(-\frac{\left[\ln\frac{S_{\tau^*}}{S} - b(\tau-\tau^*)\right]^2}{2\sigma^2(\tau-\tau^*)}\right) - \left(\frac{K+X}{S}\right)^{2b/\sigma^2}\exp\left(-\frac{\left[\ln\frac{S_{\tau^*}}{S} - 2\ln\frac{K+X}{S} - b(\tau-\tau^*)\right]^2}{2\sigma^2(\tau-\tau^*)}\right)\right],$$

$$b = r - q - \frac{\sigma^2}{2}, \tag{12a}$$

$$Q(\omega; S) = \frac{\ln\frac{K+X}{S}}{\sqrt{2\pi\sigma^2(\tau-\tau^*)^3}}\exp\left(-\frac{\left[\ln\frac{K+X}{S} - b(\tau-\tau^*)\right]^2}{2\sigma^2(\tau-\tau^*)}\right). \tag{12b}$$

The direct valuation of the expectation integrals in Eq. (11) gives

$$C(S,\tau) = Se^{-q\tau}N_2(-a_1, b_1; -\rho) - Xe^{-r\tau}N_2(-a_2, b_2; -\rho) - \left(\frac{K+X}{S}\right)^{\frac{2b}{\sigma^2}}$$

$$\times \left[ \frac{(K+X)^2}{S} e^{-q\tau} N_2(-c_1, d_1; -\rho) - X e^{-r\tau} N_2(-c_2, d_2; -\rho) \right]$$

$$+ \int_0^{\tau^*} \left\{ qS e^{-q(\tau-\tau^*+\xi)} N_2(-a_1, b_{\xi_1}; -\rho_\xi) - rX e^{-r(\tau-\tau^*+\xi)} \right.$$

$$\times N_2(-a_2, b_{\xi_2}; -\rho_\xi) - \left( \frac{K+X}{S} \right)^{\frac{2b}{\sigma^2}} \times \left[ \frac{q(K+X)^2}{S} e^{-q(\xi+\tau-\tau^*)} \right.$$

$$\left. \left. \times N_2(-c_1, d_{\xi_1}; -\rho_\xi) - rX e^{-r(\xi+\tau-\tau^*)} N_2(-c_2, d_{\xi_2}; -\rho_\xi) \right] \right\} d\xi$$

$$+ K \left[ \left( \frac{S}{K+X} \right)^{\frac{-b+\sqrt{b^2+2r\sigma^2}}{\sigma^2}} N(h) + \left( \frac{S}{K+X} \right)^{\frac{-b-\sqrt{b^2+2r\sigma^2}}{\sigma^2}} N(f) \right],$$

$$0 < S < K + X, \quad \tau^* < \tau \le T, \tag{13a}$$

where

$$a_1 = \frac{\ln \frac{S}{K+X} + b'(\tau - \tau^*)}{\sigma \sqrt{\tau - \tau^*}}, \quad a_2 = a_1 - \sigma \sqrt{\tau - \tau^*},$$

$$b_1 = \frac{\ln \frac{S}{X} + b'\tau}{\sigma \sqrt{\tau}}, \quad b_2 = b_1 - \sigma \sqrt{\tau},$$

$$c_1 = \frac{\ln \frac{K+X}{S} + b'(\tau - \tau^*)}{\sigma \sqrt{\tau - \tau^*}}, \quad c_2 = c_1 - \sigma \sqrt{\tau - \tau^*},$$

$$d_1 = \frac{\ln \frac{(K+X)^2}{SX} + b'\tau}{\sigma \sqrt{\tau}}, \quad d_2 = d_1 - \sigma \sqrt{\tau},$$

$$b_{\xi_1} = \frac{\ln \frac{S}{\widetilde{S}^*_{\tau^*-\xi}} + b'(\tau - \tau^* + \xi)}{\sigma \sqrt{\tau - \tau^* + \xi}}, \quad b_{\xi_2} = b_{\xi_1} - \sigma \sqrt{\tau - \tau^* + \xi},$$

$$\widetilde{S}^*_{\tau^*-\xi} = \widetilde{S}^*(\tau^* - \xi),$$

$$d_{\xi_1} = \frac{\ln \frac{(K+X)^2}{S\widetilde{S}^*_{\tau^*-\xi}} + b'(\tau - \tau^* + \xi)}{\sigma \sqrt{\tau - \tau^* + \xi}}, \quad d_{\xi_2} = d_{\xi_1} - \sigma \sqrt{\tau - \tau^* + \xi},$$

$$\rho = \sqrt{\frac{\tau - \tau^*}{\tau}}, \qquad \rho_\xi = \sqrt{\frac{\tau - \tau^*}{\tau - \tau^* + \xi}},$$

$$b = r - q - \frac{\sigma^2}{2}, \qquad b' = r - q + \frac{\sigma^2}{2},$$

$$h = \frac{\ln\frac{S}{K+X} + \sqrt{b^2+2r\sigma^2}(\tau-\tau^*)}{\sigma\sqrt{\tau-\tau^*}},$$
$$f = \frac{\ln\frac{S}{K+X} - \sqrt{b^2+2r\sigma^2}(\tau-\tau^*)}{\sigma\sqrt{\tau-\tau^*}}. \tag{13b}$$

(3) $K < \widetilde{S}^*(0^+) - X$

There is no voluntary early exercise by the holder and the option is recalled whenever $S$ reaches $K+X$. Hence, the value of the callable American call equals that of a European up-and-out barrier call with up-barrier $K+X$ and rebate $K$. The call value is found to be

$$\begin{aligned} C(S,\tau) &= c(S,\tau) - \left(\frac{K+X}{S}\right)^{\frac{2b}{\sigma^2}} c\left(\frac{(K+X)^2}{S},\tau\right) \\ &\quad + K\left[\left(\frac{S}{K+X}\right)^{\frac{-b+\sqrt{b^2+2r\sigma^2}}{\sigma^2}} N(z_1) + \left(\frac{S}{K+X}\right)^{\frac{-b-\sqrt{b^2+2r\sigma^2}}{\sigma^2}} N(z_2)\right], \\ & S < K+X, \quad \tau \in [0,T], \end{aligned} \tag{14}$$

where $c(S,\tau)$ is the price of the corresponding European vanilla call option and

$$z_1 = \frac{\ln\frac{S}{K+X} + \sqrt{b^2+2r\sigma^2}\tau}{\sigma\sqrt{\tau}}, \quad z_2 = \frac{\ln\frac{S}{K+X} - \sqrt{b^2+2r\sigma^2}\tau}{\sigma\sqrt{\tau}}. \tag{15}$$

### 2.3. *Resemblance between the American Capped Call and Callable American Call*

Let $C_{cap}(S,\tau;L)$ denote the value of an American capped call with a constant cap $L$. Its exercised payoff, either at expiry or upon early exercise, is specified to be

$$C_{cap}(S,\tau;L) = \max(\min(S,L) - X, 0). \tag{16}$$

Let $S^*_{cap}(\tau)$ denote the corresponding critical asset price of the American capped call. Broadie and Detemple (1995) showed that $S^*_{cap}(\tau)$ is given by

$$S^*_{cap}(\tau) = \min(\widetilde{S}^*(\tau), L), \quad \tau > 0. \tag{17}$$

If we treat $L = K+X$, it is interesting to observe that the American capped call and the callable American call (with assumptions of no notice period and full recall period) have exactly the same critical asset price boundary for all $\tau$. Following Eqs. (4a,b), the payoff at $S^*(\tau)$ for the callable American call is given by

$$C(S^*,\tau) = \begin{cases} \widetilde{S}^* - X & \text{when } S^*(\tau) = \widetilde{S}^*(\tau), \quad \widetilde{S}^*(\tau) < K+X \\ K & \text{when } S^*(\tau) = K+X, \quad \widetilde{S}^*(\tau) \geq K+X. \end{cases} \tag{18}$$

which is exactly the same as that of the American capped call as stated in Eq. (17). Hence, under the conditions of no notice period and full recall period, the callable American call has value identical to that of the American capped call.

The value of the American capped call is bounded by $L - X$ due to its payoff specification (see Eq. (16)). On the other hand, the issuer of the callable American call would choose the optimal recall strategy to limit the call value not to shoot beyond $K$. The holder of the capped call will exercise at his optimal choice once the call value reaches $L - X$, while the issuer of the callable call will recall optimally once the call value reaches $K$. Interestingly, though both types of American call options apparently have quite different contractual features, the above strategies lead to the same critical asset price boundary.

### 2.4. *Perpetual Callable American Call*

Merton (1973) analyzed the pricing model of the perpetual callable American call when the dividend yield of the underlying asset is zero. We would like to extend his analysis to the case where $q > 0$.

It is known that the upper bound of $\widetilde{S}^*(\tau)$ is given by

$$\widetilde{S}^*(\infty) = S^*_\infty = \frac{\mu}{\mu - 1} X, \tag{19a}$$

where

$$\mu = \frac{-\left(r - q - \frac{\sigma^2}{2}\right) + \sqrt{\left(r - q - \frac{\sigma}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2} > 0. \tag{19b}$$

(1) When $K > S^*_\infty - X$, the callable feature is rendered superfluous. The perpetual callable American call is identical to its non-callable counterpart, whose value is given by

$$C(S, \infty) = (S^*_\infty - X)\left(\frac{S}{S^*_\infty}\right)^\mu, \quad 0 \le S \le S^*_\infty. \tag{20}$$

(2) When $K \le S^*_\infty - X$, the perpetual call will be recalled at $S = K + X$. The perpetual callable American call becomes a perpetual barrier call with up-and-out barrier $K + X$ and rebate $K$. The call value is given by

$$C(S, \infty) = K\left(\frac{S}{K + X}\right)^\mu, \quad 0 \le S \le K + X. \tag{21}$$

## 3. Binomial Schemes

It is well known that the Cox-Ross-Rubinstein binomial scheme (Cox *et al.*, 1979) can be easily modified to price an American style security by adopting the following dynamic programming procedure at each binomial node

$$C^n = \max\left(\frac{pC_u^{n+1} + (1-p)C_d^{n+1}}{R}, S - X\right), \tag{22}$$

where $V_{roll} = \frac{pC_u^{n+1}+(1-p)C_d^{n+1}}{R}$ is the value given by the rollback and $S - X$ is the exercise payoff. Here, $p$ is the risk neutralized probability of upward jump of the asset price and $1/R$ is the discount factor over one time step. In order to incorporate the recall provision, we discuss several approaches to modify the above dynamic programming strategy.

### 3.1. *No Notice Period Requirement*

When there is no notice period requirement, the natural modification of the dynamic programming procedure to incorporate the recall feature is given by

$$C^n = \max(\min(V_{roll}, K), S - X). \tag{23}$$

The added procedure $\min(V_{roll}, K)$ compares $V_{roll}$ and $K$ to test whether the position on the issuer can be improved by recalling the option.

Recall that $S^*(\tau) \leq K + X$; and for American call, $S \leq S^*(\tau)$ in the continuation region. By observing the property: $S \leq K + X$ in the continuation region, there are only 3 possible permutations of arranging the relative magnitudes of $K$, $S - X$ and $V_{roll}$. The possible cases are: $K \geq V_{roll} \geq S - X$, $V_{roll} \geq K \geq S - X$ and $K \geq S - X \geq V_{roll}$. However, the case where $V_{roll} \geq K > S - X$ cannot occur since we have shown in Subsection 2.1 that $V_{roll} < K$ when $S < K + X$. Hence, the second case degenerates to $V_{roll} \geq K = S - X$. There are 3 possible values, $V_{roll}$, $K$ and $S - X$, which can be assumed by $C^n$ in scheme (23). Firstly, when $V_{roll}$ is taken by $C^n$, this corresponds to the case where the option is neither recalled nor exercised. This occurs when $K \geq V_{roll} \geq S - X$. Secondly, when $K$ is taken by $C^n$, the option is recalled and this corresponds to $V_{roll} \geq K = S - X$. Thirdly, when $S - X$ is taken by $C^n$, the option is exercised prematurely by the holder. This occurs when $K \geq S - X \geq V_{roll}$. These three scenarios are exhaustive and mutually exclusive.

Alternatively, another possible choice of the dynamic programming procedure is given by

$$C^n = \min(\max(V_{roll}, S - X), \max(K, S - X)). \tag{24}$$

The term $\max(V_{roll}, S - X)$ represents the optimal strategy of the holder, given no recall of the option. On the other hand, upon recall of the option, the holder chooses to accept the recall price $K$ or to exercise the call, so the payoff is given by $\max(K, S - X)$. The issuer chooses to recall or to abstain from recalling to minimize the option value with reference to the possible actions of the holder; and consequently, the option value is given by Eq. (24).

Furthermore, by observing $S \leq K + X$ in the continuation region, we have $\max(K, S - X) = K$. The above equation can be simplified to become

$$C^n = \min(K, \max(V_{roll}, S - X)). \tag{25}$$

It can be shown that the values taken by $C^n$ using scheme (25) under the above 3 cases: $K \geq V_{roll} \geq S - X$, $V_{roll} \geq K = S - X$ and $K \geq S - X \geq V_{roll}$, agree exactly with those obtained using scheme (23).

On the other hand, the binomial scheme for the calculation of the value of the American capped call is given by

$$C^n = \max(V_{roll}, \min(S, L) - X). \tag{26}$$

Suppose we treat $L = K + X$, it is observed that the values taken by $C^n$ using scheme (26) under the same 3 cases on relative magnitudes of $V_{roll}$, $K$, $S - X$ are identical to those obtained using schemes (23) and (25).

In summary, though the above 3 dynamic programming strategies (as depicted in schemes (23), (25) and (26)) are developed based on different financial arguments and take quite different forms, they generate the same numerical values *in the continuation region* of the option model under identical payoff conditions.

### 3.2. *Presence of Notice Period Requirement*

Suppose there exists notice period requirement, the only required modification to the binomial schemes (23) and (25) is the replacement of the recall price $K$ by $c_n(S, t_n)$, where $c_n(S, t_n)$ is the value of the vested European option with terminal payoff defined in Eq. (5) and $t_n$ is the length of the notice period. Accordingly, we may use

$$C^n = \min(c_n(S, t_n), \max(V_{roll}, S - X)), \tag{27}$$

to price the corresponding American callable call with notice period requirement.

### 3.3. *Numerical Experiments*

We have verified by numerical experiments that the three binomial schemes (23, 25, 26) give identical numerical values for the American callable option values in the continuation region. The numerical option values are also compared with the value obtained from the numerical valuation of the analytical price formula (13) (see Table 1). The parameter values used in the sample calculations are: $r = 0.1$ (per annum), $q = 0.08$ (per annum), $\sigma = 0.3$ (per annum), $X = 1.0$, $K = 0.5$, $T = 2.0$ (years); and the number of time steps used in the binomial schemes is 100. It is seen that the call values obtained using the binomial schemes and the analytical price formula agree favorably well (within tolerance of numerical accuracy). This serves as a verification of the validity of the theoretical arguments used in deriving the analytical price formula and the construction of the binomial schemes.

As a cautious note, numerical accuracy of the binomial schemes can be improved by placing the binomial nodes along the up-barrier: $S = K + X$.

*Table 1.* Comparison of the callable American call option values obtained using binomial scheme (25), and numerical valuation of the analytical price formula (13). The parameter values of the option model are: $r = 0.1$ (per annum), $q = 0.08$ (per annum), $\sigma = 0.3$ (per annum), $X = 1.0$, $K = 0.5$, $T = 20$ (years), and the number of binomial steps is 100. The two sets of call values agree favorably well.

| Asset price $S$ | Option values by price formula (13) | Option values by scheme (25) |
|---|---|---|
| 0.50 | 0.0069 | 0.0069 |
| 0.75 | 0.0535 | 0.0534 |
| 1.00 | 0.1601 | 0.1598 |
| 1.25 | 0.3131 | 0.3150 |

## 4. Pricing Behaviors under Different Conditions on Recall Provision

In this section, we present the plots of the critical asset price boundaries $S^*(\tau)$ and option values of the callable American call option under different scenarios (full or partial period of recall provision, and finite or zero notice period). These results were obtained from the numerical calculations using the binomial schemes (25) and (27). In the numerical calculations, we adopt the *same set of option parameter values* as used in Table 1.

First, we consider the case where the recall provision is activated throughout the whole life of the option, $0 \leq \tau \leq 2.0$. In Figure 1(a), we plot the option value against $\tau$ at asset value $S = 1.0$. The dotted line shows the asymptotic option value at infinite time to expiry [corresponds to the value of the perpetual option given in Eq. (21)]. The call option value is seen to be a monotonically increasing function of $\tau$ with an upper bound. Next, we show the plots of option values against $S$ at varying time to expiry in Figure 1(b). The upper price curve (solid) corresponds to $\tau = 1.0$, the middle price curve (dashed) corresponds to $\tau = 0.29$ [this is the critical value $\tau^*$ obtained by solving Eq. (9)], and the lower price curve (dotted) corresponds to $\tau = 0.2$. At $\tau \geq \tau^* = 0.29$, the option value is seen to stay below $K$ (which equals 0.5) at asset value below $S^*(\tau)$ (which equals $K + X = 1.5$). At $\tau = 0.2$, the callable American call is exercised prematurely at $S^*(\tau) = 1.38$ (which is less than $K + X = 1.5$). In the stopping region where $S \geq S^*(\tau)$, and at any value of $\tau$, the option value is equal to $S - X$. The plotted critical asset price boundary $S^*(\tau)$ against $\tau$ as shown in Figure 1(c) reveals the agreement with the theoretical prediction on $S^*(\tau)$ as stated in Eq. (10b). We observe that $S^*(\tau)$ increases monotonically with $\tau$ until $\tau^* = 0.29$, then stays at the constant value $K + X$ (which equals 1.5) at $\tau \geq \tau^*$.

In Figure 2, we show the plots of the critical asset prices for the callable American call (solid curve) and its non-callable counterpart (dotted curve), where the recall provision is activated in the last part of the option's life, $0 \leq \tau \leq 0.5$. Note that $S^*(\tau)$ is given by $\min(\widetilde{S}^*(\tau), K + X)$ during the recall provision period. At times prior to the recall provision period (corresponds to $\tau > 0.5$), though the issuer has not been given the recall privilege,

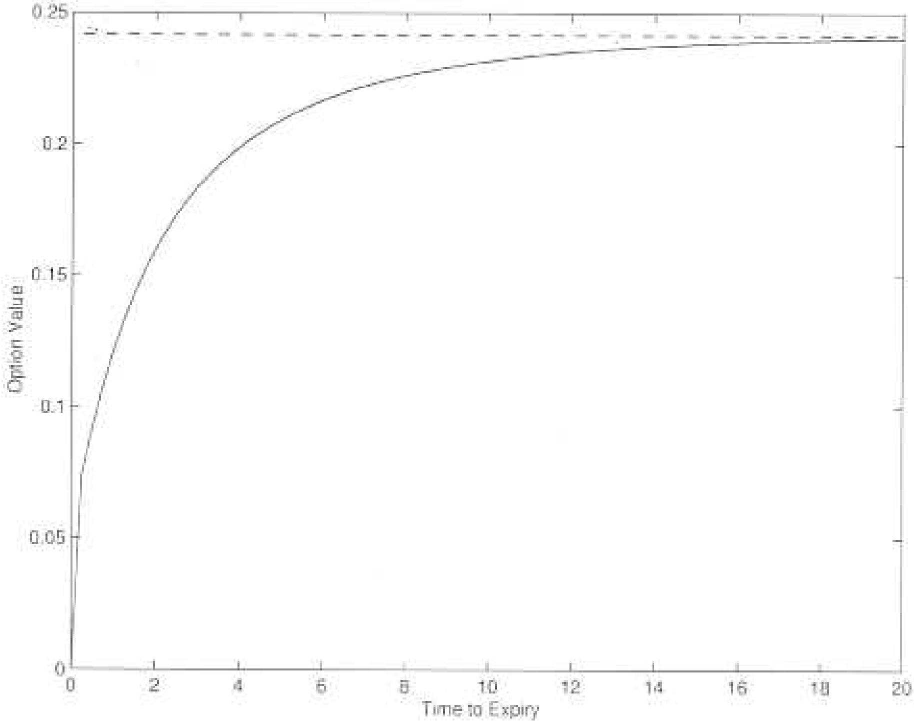


*Figure 1(a).* The recall provision is activated throughout the whole option's life, $0 \leq \tau \leq 2.0$. The call value evaluated at $S = 1.0$ is plotted against $\tau$. The call value is shown to be a monotonically increasing function of $\tau$ bounded above by the value of the perpetual call (shown in dotted line).

it may become optimal for the holder to exercise the American call prematurely at a critical asset price lower than $\widetilde{S}^*(\tau)$. The imposition of the recall provision starting at $\tau = 0.5$ is anticipated during the time period $\tau > 0.5$; so the optimal early exercise policy at $\tau > 0.5$ is adopted such that the option value and $S^*(\tau)$ remain to be continuous at $\tau = 0.5$.

Next, we analyze the impact on option value and $S^*(\tau)$ when the recall provision period is limited to the earlier part of the option's life, $1.5 \leq \tau \leq 2.0$. In Figure 3(a), we plot the option value against $\tau$ corresponding to $S = 1.5$ (upper curve) and $S = 1.48$ (lower curve). Within the period $0 \leq \tau < 1.5$, the option value is the same as that of the non-callable American call. The option value may go above the recall price $K$. Within the period $1.5 \leq \tau \leq 2.0$, the option will be recalled at $S = 1.5$, and the option value then becomes $K = 0.5$. This explains why the upper curve (corresponds to $S = 1.5$) assumes the constant value 0.5 during $1.5 \leq \tau \leq 2.0$ (the option is being recalled). On the other hand, at the asset price level $S = 1.48$, the option remains alive throughout the time period $0 \leq \tau \leq 2.0$. The option value shown in the lower curve (corresponds to $S = 1.48$) is seen to be continuous across $\tau = 1.5$. Unlike the option value which remains continuous across $\tau = 1.5$, the critical asset price $S^*(\tau)$ exhibits jump at $\tau = 1.5$ (see Figure 3(b)). During the period

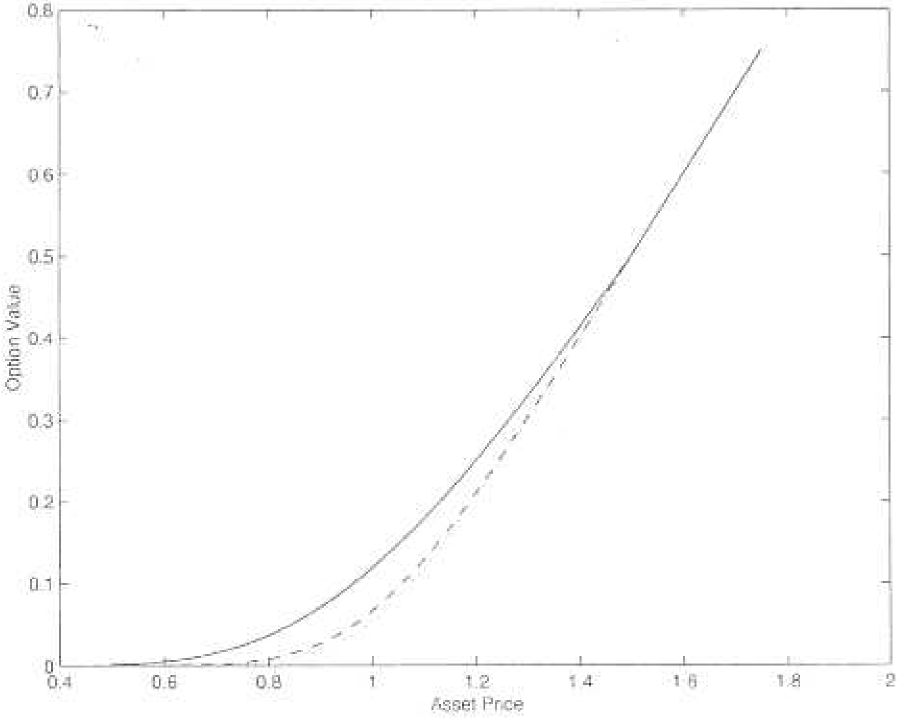


*Figure 1(b).* We use the same recall period as that of Figure 1(a). The call value is plotted against $S$ at varying value of $\tau$. The critical value $\tau^*$ [as obtained by solving Eq. (9)] is equal to 0.29. The upper solid curve corresponds to $\tau = 1.0$ (above $\tau^*$), the middle dashed curve corresponds to $\tau = 0.29$ (equal $\tau^*$) and the lower dotted curve corresponds to $\tau = 0.2$ (below $\tau^*$). When $\tau \geq \tau^*$, the critical asset value $S^*(\tau) = 1.5$ (equals $K + X$); and when $\tau = 0.2$, $S^*(0.2) = 1.38$. The call value in the stopping region, where $S \geq S^*(\tau)$, is always equal to $S - X$.

where the recall provision is not activated, $0 \leq \tau < 1.5$, we have $S^*(\tau) = \widetilde{S}^*(\tau)$ since this call option behaves like its non-callable counterpart. At the time instant right after the lifting of the recall provision (corresponds to $\tau = 1.5^-$), the asset price at which the option value equals $K = 0.5$ is found to be 1.48 (which is less than $K + X = 1.5$). The issuer should recall the option at $S = 1.48$ at the exact time instant $\tau = 1.5$ the option value will be above the recall price at $S > 1.48$ if unrecalled. During the time period $1.5 < \tau \leq 2.0$, the option will be recalled at the critical asset price $S^*(\tau) = K + X = 1.5$. In summary, the critical asset price is given by

$$S^*(\tau) = \begin{cases} \widetilde{S}^*(\tau) & 0 \leq \tau < 1.5 \\ 1.48 & \tau = 1.5 \\ K + X & 1.5 < \tau \leq 2.0. \end{cases}$$

The dotted and solid curves in Figure 4(a) show the option values against $S$ at $\tau = 0.76$ with and without the notice period requirement, respectively. The length of the notice

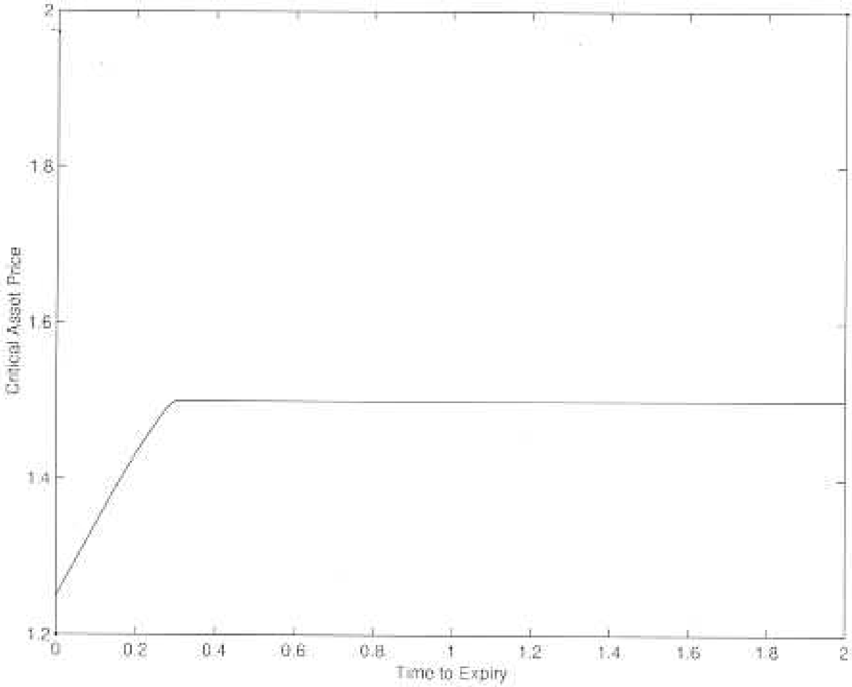


*Figure 1(c).* We use the same recall period as that of Figure 1(a). The critical asset price $S^*(\tau)$ is plotted against $\tau$, where it increases monotonically with $\tau$ until $\tau = 0.29$, then stays at the constant value 1.5 at $\tau \geq 0.29$.

period is chosen to be $1/12$ (year). The critical asset prices corresponding to zero and $1/12$ (year) notice period are found to be 1.5 and 1.73, respectively; and the option value with the presence of notice period is higher than those without the notice period. Figure 4(b) shows the plot (solid curve) of the critical asset price $S^*(\tau)$, where $S^*(\tau)$ increases monotonically from $\tau = 0$ to $\tau = 0.76$, then decreases monotonically from $\tau = 0.76$ onward. The maximum value of $S^*(\tau)$, denoted by $S^*_{\max}(t_n)$, occurs at the asset price at which the intrinsic value $S - X$ equals the value of the vested European option upon recall. The critical value $\widehat{\tau}$ at which $S^*(\tau)$ achieves its maximum value is obtained by solving the algebraic equation: $\widetilde{S}^*(\tau) = S^*_{\max}(t_n)$, where $S^*_{\max}(t_n)$ is the solution to another algebraic equation: $S - X = c_n(S, t_n)$. When $\tau > \widehat{\tau}$, the option is terminated prematurely due to optimal recall by the issuer. The critical asset price to recall is given by the condition: $C(S, \tau) = c_n(S, t_n)$, that is, the issuer should recall whenever the option value reaches $c_n(S, t_n)$. The critical asset value to recall is seen to be a decreasing function of $\tau$. In summary, over the time period $0 \leq \tau < \widehat{\tau}$, $S^*(\tau)$ is monotonically increasing with $\tau$ and the American call will be early exercised by the holder when $S$ hits $S^*(\tau)$; over the time period $\tau \geq \widehat{\tau}$, $S^*(\tau)$ is monotonically decreasing with $\tau$ and the American call will be recalled by the issuer when $S$ reaches $S^*(\tau)$. Lastly, the dependence of $S^*_{\max}(t_n)$ on the length of notice period $t_n$ is revealed in Figure 4(c). It is observed that $S^*_{\max}(t_n)$ is an

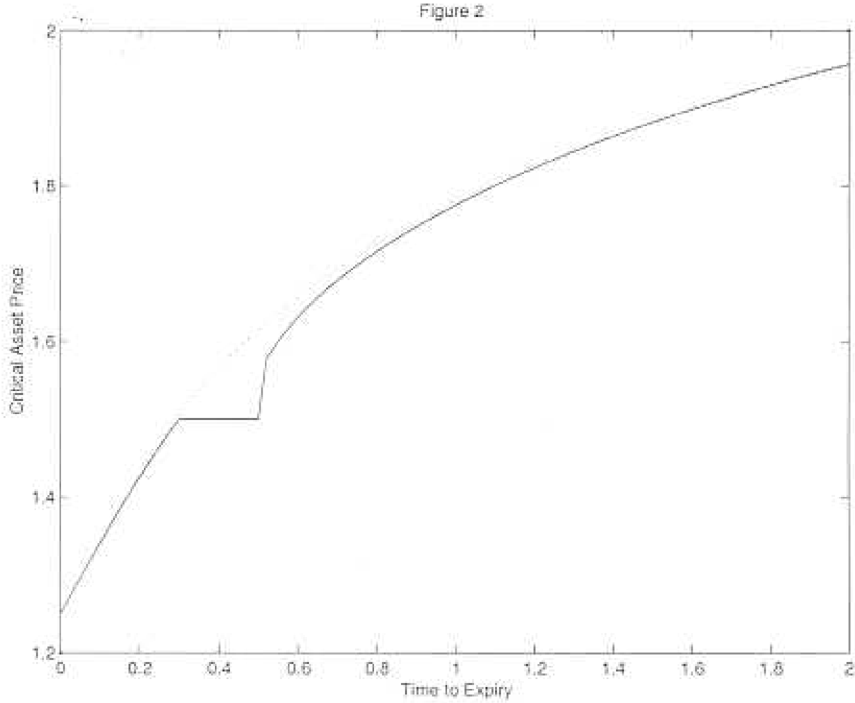


*Figure 2.* The critical asset price $S^*(\tau)$ (solid curve) is plotted against $\tau$, where the recall provision is activated during the later part of the option's life $(0 \leq \tau \leq 0.5)$. The critical asset price $\tilde{S}^*(\tau)$ for the non-callable counterpart (dotted curve) is also plotted for comparison. With the anticipation of the imposition of the recall provision starting at $\tau = 0.5$, the call option may be exercised optimally at a lower critical asset price compared to its non-callable counterpart during the time period $\tau > 0.5$.

increasing function of $t_n$.

Table 2 lists the values of the callable American call at two asset price levels and with varying length of notice period. The parameter values used in the option model are identical to those used in Table 1. It is seen that there are noticeable increases in call value from no notice period to short notice period (5 days), but the rate of increase drops with longer length of notice period. The call value can be higher than the recall price $K$ (which equals 0.5) when the notice period requirement is present.

## 5. Summaries and Conclusions

By using the callable American call option as an example, we investigate fully the effects of the callable feature on the early exercise policy of American style derivatives, including the consideration of the impact of the notice period requirement and partial recall period. When the recall price $K$ is sufficiently high, the callable feature is rendered su-

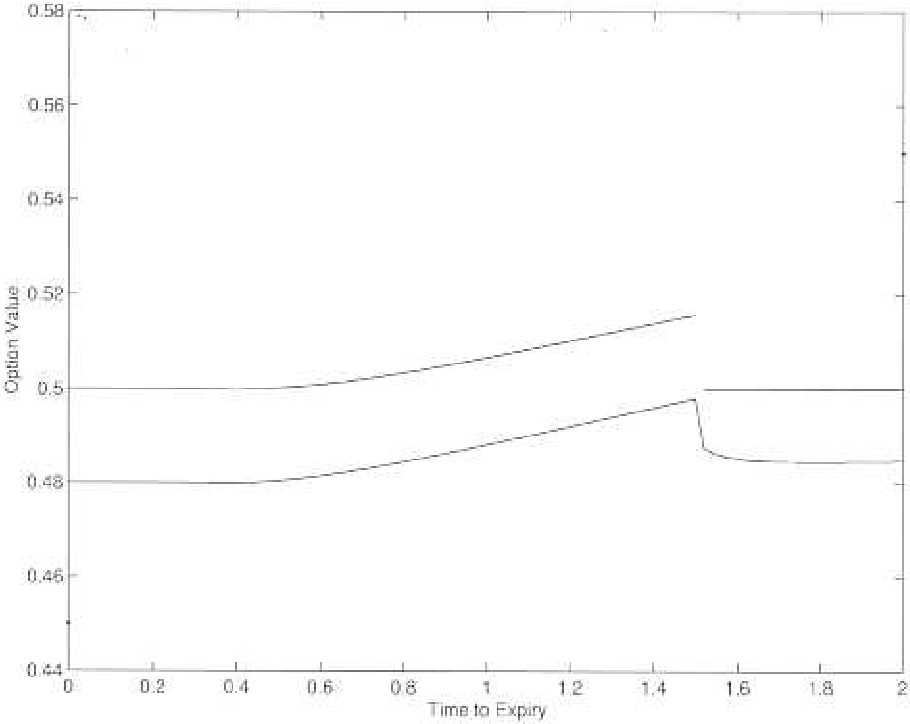


*Figure 3(a).* The recall provision is activated within the period $1.5 \le \tau \le 2.0$. The option value is plotted against $\tau$ at $S = 1.5$ (upper curve) and $S = 1.48$ (lower curve). The option value function evaluated at $S = 1.5$ has a jump at $\tau = 1.5$.

*Table 2.* We examine the impact of varying length of notice period on the value of the callable American call at two asset price levels. The rate of increase of the call value with increasing length of notice period is more significant at short length of notice period. The call value can be higher than the recall price when the notice period requirement is present. Here, we use the same set of parameter values as those in Table 1.

| Notice period (days) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| Asset price $S = 1.0$ | 0.1598 | 0.1618 | 0.1620 | 0.1621 | 0.1622 | 0.1622 | 0.1623 | 0.1623 | 0.1623 |
| Asset price $S = 1.48$ | 0.4845 | 0.5004 | 0.5031 | 0.5045 | 0.5054 | 0.5059 | 0.5064 | 0.5067 | 0.5069 |

perfluous and the callable American option is reduced to its non-callable counterpart. The callable American call option may be terminated prematurely either by forced conversion (under the optimal recall policy of the issuer) or voluntary early exercise (under the optimal early exercise policy of the holder) when the asset price rises to some critical price level.

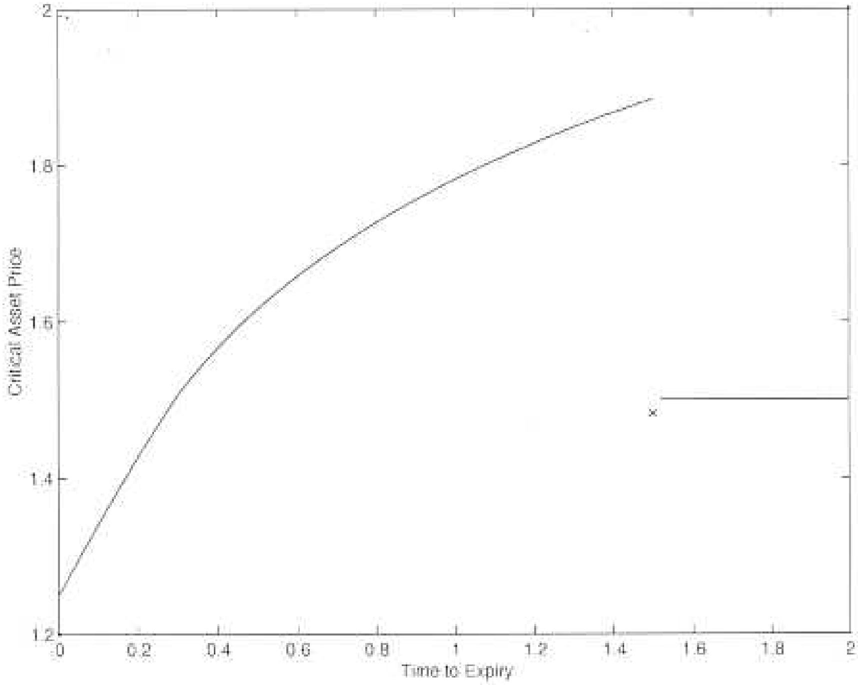


*Figure 3(b).* The same recall period is used as that in Figure 3(a). The critical asset price $S^*(\tau)$ exhibits jumps at $\tau = 1.5$. During the period $0 \leq \tau < 1.5$, $S^*(\tau)$ is the same as that of the non-callable American call option. At the exact instant $\tau = 1.5$, the option will be recalled at $S = 1.48$. During the period $1.5 < \tau \leq 2.0$, $S^*(\tau)$ stays at the constant value 1.5.

First, we consider the case where there is no notice period requirement. During the time period where the recall provision is activated, the critical asset price $S^*(\tau)$ behaves as a monotonically increasing function of $\tau$, then assumes some constant value. The time varying part of $S^*(\tau)$ occurs during the time interval where the American call is terminated prematurely due to the optimal early exercise by the holder, while the constant value part of $S^*(\tau)$ corresponds to the critical asset price at which the call option is optimally recalled by the issuer. This constant value of $S^*(\tau)$ is equal to the sum of the recall price $K$ and the strike price $X$. During the time period when the recall provision is activated, the critical asset price $S^*(\tau)$ is given by the minimum of $K + X$ and $\widetilde{S}^*(\tau)$, where $\widetilde{S}^*(\tau)$ is the optimal exercise boundary of the non-callable counterpart.

We also analyze the impact of anticipated lifting and imposition of the recall provision. Suppose the recall provision is active now but will be lifted in the next instant. Right at the last time instant when the recall provision is still active, the American call may be recalled at a critical asset price which falls below $K + X$. At the next moment when the recall provision is lifted, the critical asset price may encounter an upward jump. However, the American call value remains continuous across the instant of lifting of recall provision. On

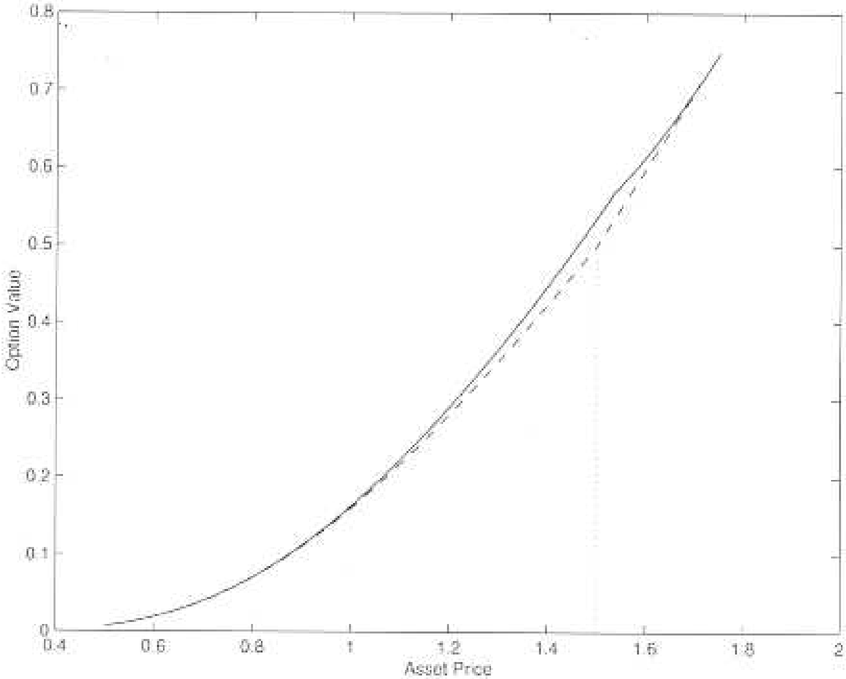


*Figure 4(a).* The figure shows the plots of the option value against $S$ at $\tau = 0.76$ of a callable American call with zero notice period (dotted curve) and with notice period of 1/12 year (solid curve).

the other hand, when the imposition of the recall provision is anticipated in the near future, the holder may exercise the callable American call optimally at a lower critical asset price compared to the non-callable counterpart.

With the presence of the notice period requirement, the critical asset price at which the mode of termination of the American call changes from early exercise to forced conversion occurs at the asset value where the intrinsic value $S - X$ equals the value of the vested European option upon recall. This particular critical asset price is the maximum among all critical asset prices throughout the option's life. Also, this maximum critical asset price increases with increasing length of the notice period. Further, the critical asset price at which the American call is optimally recalled by the issuer decreases with increasing $\tau$. This is in contrast to the case of zero notice period where the critical asset price to recall is equal to the constant value $K + X$.

There are three quantities that determine the optimal calling and exercising policies, namely, the recall price $K$, intrinsic value $S - X$, and rollback value $V_{roll}$. Three numerical algorithms have been proposed for the numerical valuation of the option value. The binomial algorithm, with the adoption of the usual dynamic programming procedure at each node, has to be modified accordingly in order to incorporate the recall feature. In one binomial scheme, the minimum between $V_{roll}$ and $K$ is first chosen at each node, then the higher

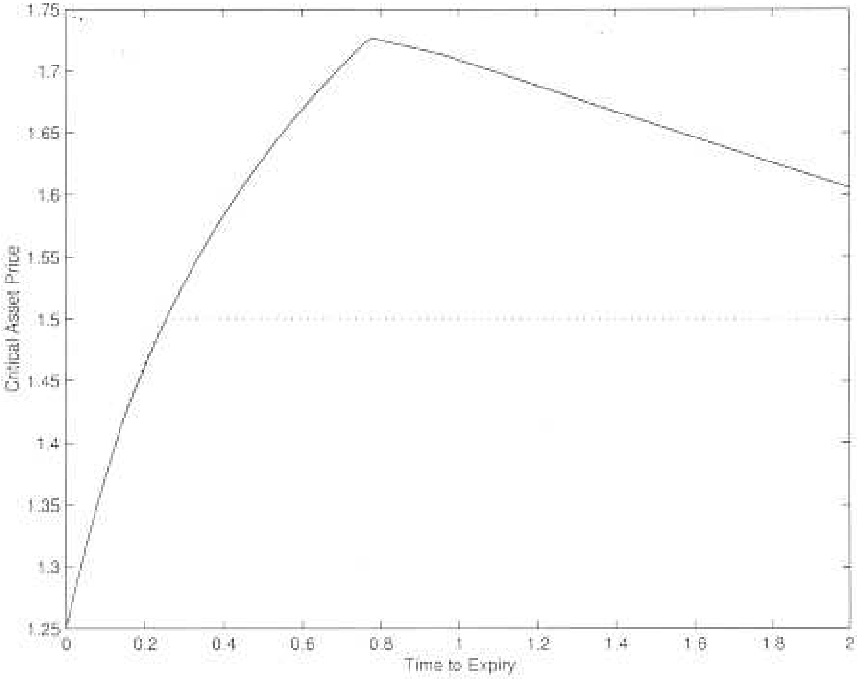


*Figure 4(b).* The figure compares the critical asset price $S^*(\tau)$ corresponding to zero notice period (dotted curve) and notice period of 1/12 year (solid curve). With the presence of notice period, $S^*(\tau)$ first increases with increasing $\tau$, reaches a maximum then decreases.

value between that minimum and $S - X$ is taken. The other binomial scheme sets the cap $K$ on the maximum between $V_{roll}$ and $S - X$. The third binomial scheme chooses the minimum between $S - X$ and $K$ first, then takes the higher value between that minimum and $V_{roll}$. Though these three binomial schemes are constructed based on different financial arguments, it has been verified through numerical experiments and justified by theoretical arguments that they generate the same numerical option values for the callable American call option in the continuation region.

It is interesting to observe that the pricing behaviors of the callable American call (under the assumptions of no notice period requirement and full period of recall provision) and the American capped call are identical, though apparently the two types of American call options have quite different contractual features.

The analysis of the effects of the callable feature on the American call option model may shed light on the better understanding of the impact of the callable feature on other financial instruments with the early exercise or conversion right, like convertible bonds.

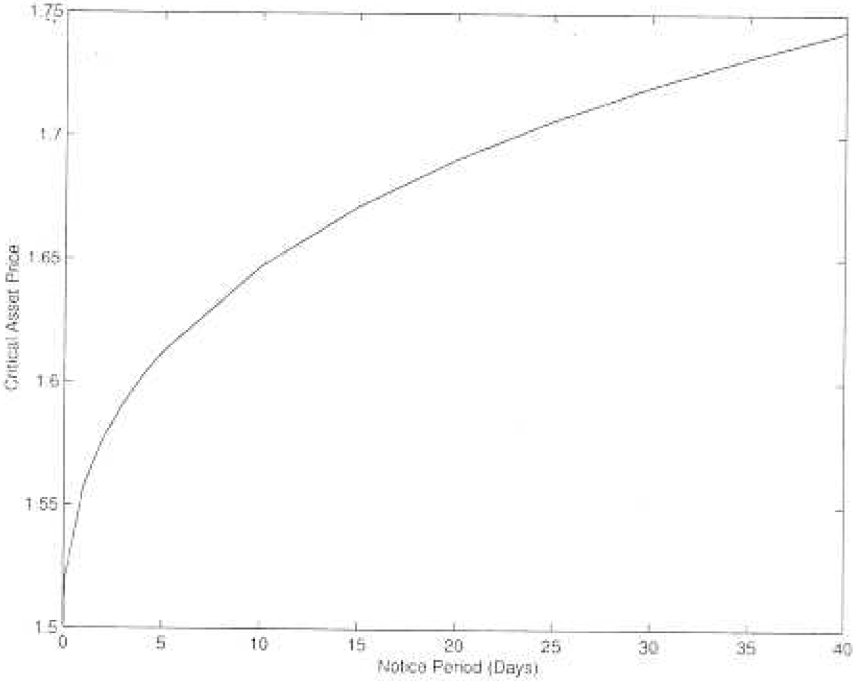


*Figure 4(c).* With the presence of notice period requirement, the maximum critical asset price $S^*_{max}(t_n)$ is obtained by solving the equation: $S - X = c_n(S, t_n)$. The figure shows that $S^*_{max}(t_n)$ is a monotonically increasing function of the length of the notice period $t_n$.

## Acknowledgments

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