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Contingent claim approach for analyzing the creditrisk of defaultable currency swaps

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ABSTRACT. In this paper, we analyze the credit risk associated with defaultable currency swaps under the contingent claim analysis framework. One of the swap parties is subject to intertemporal default risk while the other swap party is assumed to be default free. The event triggering the intertemporal swap default is endogenized by characterizing the creditworthiness of the defaultable swap party. The firm value is used to determine whether the defaultable swap party can fulfil the cashflows associated with the swap contract and other financial obligations. The impact of various clauses and settlement rules in the currency swap contracts are examined. The influences of the rate risk on the swap rates, as exemplified by fluctuating volatilities of the exchange rate, varying correlation between firm value and exchange rate, are also analyzed.

1. INTRODUCTION

In simple terms, a financial swap is the exchange of cashflow or asset based on an underlying index under some prescribed terms in the swap contract. The growth of the swap markets has been phenomenal since the first swap contract structured in 1981. There are a number of explanations for the popularity of swaps. Swaps allow firms to exploit market inefficiencies and to arbitrage quality spread differentials. Also, swaps may be used to adjust the repricing interval of a firm's assets or liabilities in order to reduce the interest rate risk or the exchange rate risk.

Since any swap involves mutual obligations to exchange cashflows, default occurs if a counterparty owes a payment and becomes insolvent. The two major risks of swap contracts are the rate risk and the credit risk. The rate risk arises from the change in the interest rate or the exchange rate, while the credit risk occurs since either swap party may default. Unlike debt contracts, the swap default risk is two-sided since it depends on the realized path of the underlying index of the swap which can result in either party to make payments at different times in the life of the swap.

The maximum loss associated with the credit risk is measured by the swap's replacement cost. The question is: how much spread is appropriate to cover the swap credit risk between the higher-rated and lower-rated firms. Since the credit

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risk borne by swap dealers is far smaller in proportion to the principal than in debt contracts, it is meaningless to estimate the spreads on higher-rated and lower-rated swaps from the qualify spread on bonds of similar credit categories. There are other unique features regarding contract clauses and settlement rules in swap contracts that may affect the credit risk of swaps. For example, the limited feature of a swap discharges all the non-defaulting counterparty's obligations in the event of default, and the settlement can be net or gross, the swaps can be junior, pari-passu or senior than debts and other liabilities.

Hazard rate approach and contingent claim approach

In the literature, there are two popular approaches for pricing the credit spreads of risky financial instruments. These two approaches are commonly called the hazard rate approach and the contingent claim approach (or the firm value approach). The hazard rate models specify exogenously the processes for bankruptcy and the payoff on the risky instruments conditional on default. The model introduces credit classes, with transition from one class to another driven by a deterministic credit migration matrix to account for credit ratings behaviors. Jarrow and Turnbull (1997) and Nielsen and Ronn (1997) applied the hazard rate approach to evaluate the impact of default risk on risky swaps. The hazard rate model views financial instruments as members from a credit rating class, and so it loses sight of the specific features associated with the issuing firm and contractual terms in the instrument. If we would like to investigate the impact of some specified contractual terms and settlement clauses in swap contracts, then it would be more preferable to follow the contingent claim approach (or the firm value approach).

Previous works on default risk analysis of swaps

It has been observed in the financial markets that swap rates seem not to depend sensibly on the credit ratings of the counterparties (Litzenberger, 1992). One of the earlier works on the use of the contingent claim approach in default analysis of swaps is the risk model by Cooper and Mello (1991). Their model assumes a simple capital structure, where one of the swap counterparties is risky and the other is riskless. The firm of the risky party also has debt outstanding. The remainder of the risky firm is financed with equity. Both the debt and swap are assumed to mature on the same date, and the swap is junior in claims than the debt. Cooper and Mellon's model is later extended by Baz and Pascutti (1996) where various swap clauses are considered. They considered swap covenants along three dimensions: settlement of swap amounts can be net or gross, seniority regimes for swaps relative to debt can be senior, pari-passu or junior, and counterparty obligations under insolvency. Li (1998) developed a more refined firm value model where swaps are valued as contingent claims which pay a stream of cash flow equal to the net payments between the swap counterparties.

In the above swap default analysis models, the reorganization of the counterparties when their firm values fall below some threshold values and possible deviations from the strict priority rules have been neglected. At the next level of sophistication of credit risk models, the defaultable swap models should allow intertemporal default risk of the counterparties prior maturity of the swap contract.

Scope of the paper

In this paper, the contingent claim approach is used to construct an equilibrium model for analyzing default risk of currency swaps, where the sources of uncertainty in the models are the rate distribution of the exchange rate, and the firm value of one of the swap parties. For simplicity of analysis, we assume that only one of the swap parties is defaultable. The default risk and the exchange rate risk, and their interaction are analyzed in a combined framework in the contingent claim models, rather than being artificially decomposed as in other approaches. The success of the contingent claim analysis method relies on the precise prescription of the processes that lead to financial distress of the swap parties and the bankruptcy terms upon default. The description of these default conditions are translated into the auxiliary conditions of the contingent claim models. The pricing behaviors of currency swaps with intertemporal default possibilities are examined critically. In particular, special considerations are directed to the swap rate spreads with respect to different parameter values. The set of parameters include the variance rates of the firm value process and the exchange rate process, the correlation coefficient between the processes, and the threshold level of default.

2. FORMULATION OF DEFAULTABLE CURRENCY SWAP MODELS

The option pricing theory has been shown to provide a universal valuation framework for contingent claims. The contingent claim approach on the risk analysis of corporate debts, initiated by Merton (1974), represents an elegant use of the option pricing theory in corporate finance, where corporate liabilities can be viewed as combinations of option contracts. The various claims (such as debt and swap contracts) are modelled in terms of appropriate auxiliary conditions in the contingent claim models. The riskiness of a financial instrument is measured by its risk premium, which is the spread between the prices of the risky instrument and its non-risky counterparty.

Later enhancements of the contingent claim analysis of credit risk include the possibilities of early default, where the firm defaults when the value of the firm's assets falls to a lower threshold value, or called the reorganization boundary (Black and Cox, 1976). Various formulations of default-triggering mechanisms have been developed in the literature. This line of research has been continued with various refinements in the definition of the reorganization boundary, for example, in the recent works by Longstaff and Schwartz (1995), Rich (1996), Briys and de Varenne (1997). Anderson and Tu (1998) proposed the strategic contingent claims models where the reorganization of the firm is also dependent upon actions taken either by the debtors or creditors. These methodologies, though mainly developed for the valuation of risk premia of corporate bonds, can also be adopted to the risk analysis models of swaps.

2.1 Assumptions in the defaultable currency swap models

A currency swap involves exchange of principals and interest payments in two currencies. The counterparties of a currency swap are companies in two different countries; say, let company A be domestic and company B be foreign. We assume company A to be risky and company B to be default free. The firm value F of company A is assumed to follow the stochastic process

$$dF = \mu_F F\ dt + \sigma_F F\ dZ_F \quad (2.1)$$

where μ_F is the instantaneous expected rate of return on the firm's assets, σ_F^2 is the instantaneous variance rate and dZ_F is the standard Wiener process. Further, it is assumed that the value of the firm is independent of the capital structure of the firm. Also, the value of the swap contract is assumed to be negligibly small compared to the firm value F.

Following the intertemporal default model proposed by Longstaff and Schwartz (1995), we assume that there is constant threshold value H for the firm value below which financial distress occurs. That is, the firm value F must be greater than H in order that the firm continues to be able to meet its contractual obligations. The exchange rate S, which is defined as the domestic currency price of one unit of foreign currency, is also assumed to follow the lognormal process

$$dS = \mu_S S \; dt + \sigma_S S \; dZ_S. \quad (2.2)$$

where μ_S and σ_S^2 are the expected rate of return and variance rate of the exchange rate, respectively.

Cashflows between currency swap counterparties

The domestic company A has comparative advantage in borrowing domestic loan but it wants to raise foreign capital, while situations for the foreign company B happen to be reverse to those of company A. It then becomes natural for both companies to enter into a currency swap so as to exploit the comparative advantages in borrowing rates. Let P_d and P_f denote the principals in domestic and foreign currencies, respectively, where $P_d = S_0 P_f$ and S_0 is the exchange rate at the beginning of the swap contract. When the currency swap is initiated, company A pays to company B the principal P_d, in exchange for receiving P_f. During the period of the swap contract, A makes interest payments to B in foreign currency since A has received P_f from B; and vice versa, B makes domestic interest payments to A. For simplicity of analysis, we assume that these payments are in continuous streams at the constant swap rates c_d and c_f, that is, A pays $c_f P_f$ continuously to B while B pays $c_d P_d$ continuously to A throughout the whole life of the swap. The swap rates are chosen such that the value of the swap contract is set to be zero at the beginning of the contract. At maturity of the swap, company A exchanges P_f for P_d with company B. The cashflows between the two currency swap counterparties are summarized in Figure 1.

Settlement rules

When the firm value F of company A falls to the threshold value H, company A is forced to reorganize. The settlement payment to the swap counterparty upon intertemporal default depends on the settlement clauses in the swap contract. Under the full (limited) two-way payment clause, the non-defaulting counterparty is required (not required) to pay if the final net amount is favorable to the defaulting party. When company A defaults and the swap is favorable to its counterparty, company B will receive only the fraction $1 - w$ of the market quotation value of the swap agreement. Here, w denotes the proportion of write-down upon default, and the market quotation value refers to the value of the corresponding riskless swap contract.

2.2 Governing equations

Let $v(S,t)$ and $V(S,F,t)$ denote the values of the riskless and defaultable currency swap contracts to company B, respectively. The governing equations for $v(S,t)$ and $V(S,F,t)$ are derived using the standard riskless hedging argument, and the auxiliary conditions are developed by modelling the cashflow settlements at maturity and upon intertemporal default.

2.2.1 Riskless currency swap models

The governing equation for $v(S,t)$ is given by

$$\frac{\partial v}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 v}{\partial S^2} + (r_d - r_f)\, S \frac{\partial v}{\partial S} + (P_f c_f S - P_d c_d) - r_d v = 0, \qquad 0 < S < \infty, \quad t > 0, \tag{2.3}$$

where r_d and r_f are the domestic and foreign riskless interest rates, respectively. At maturity, the payoff is

$$v(S,T) = P_f S - P_d. \tag{2.4}$$

For a given domestic swap rate c_d, the foreign swap rate c_f is chosen such that the value of the riskless swap agreement is zero at the initiation of the agreement.

2.2.2 Defaultable currency swap models

The governing equation for $V(S,F,t)$ is given by

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + \rho_{SF}\sigma_S\sigma_F SF \frac{\partial^2 V}{\partial S \partial F} + \frac{\sigma_F^2}{2} F^2 \frac{\partial^2 V}{\partial F^2} \\ +[r_d F - (P_f c_f S - P_d c_d)]\frac{\partial V}{\partial F} + (r_d - r_f) S \frac{\partial V}{\partial S} + (P_f c_f S - P_d c_d) - r_d V = 0, \\ 0 < S < \infty, H < F < \infty, t > 0,\end{aligned} \tag{2.5}$$

where ρ_{SF} is the correlation coefficient between S and F. The prescription of the auxiliary conditions depends on the settlement clauses of the swap agreement upon default. The limited and full two-way settlement clauses are both considered here.

Limited two-way settlement

(i) At maturity, the two counterparties exchange their principals when A is non-defaulting. When A becomes default, B receives $1 - w$ of $P_f S_T - P_d$ from A if $P_f S_T - P_d > 0$, but B pays nothing to A if $P_f S_T - P_d \le 0$.

$$V(S,F,T) = \begin{cases} P_f S - P_d, & \text{F>H} \\ (1-w)\max(P_f S - P_d, 0), & \text{F = H} \end{cases}. \tag{2.6a}$$

(ii) When the firm value F tends to infinity, it is almost sure that F will not fall below H at subsequent times; hence,

$$\lim_{F\to\infty} V(S,F,t) = v(S,t), \quad \text{for all } t. \tag{2.6b}$$

(iii) When the firm value F drops to the threshold value H, company A becomes default. The counterparty B pays nothing if the swap is favorable to the defaulting party A; otherwise, it receives $1-w$ of the value of the corresponding riskless swap contract. Hence,

$$V(S,H,t) = (1-w)\max(v(S,t),0), \quad \text{for all } t. \tag{2.6c}$$

(iv) When the exchange rate S drops to zero, it will stay at that level at all subsequent times. The foreign payments becomes worthless, and the swap contract behaves like a bond where company B pays the continuous payments $c_d P_d$ and final par value P_d to company A. The present value of these payments equals to $P_d \left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\}$, provided that company A remains non-defaulting. When company A defaults, company B automatically stops the payments due to the limited two-way settlement clause. Hence,

$$V(0,F,t) = -P_d \left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\} P[F \ge H], \tag{2.6d}$$

where $P[F \geq H]$ denotes the probability that the firm value F stays above H for the whole life of the swap contract. It is known that

$$P[F \geq H] = N\left(\frac{\ln\frac{V}{H}+\left(r_d-\frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_F\sqrt{T-t}}\right) - e^{-\frac{2r_d}{\sigma_V^2}\ln\frac{V}{H}}N\left(\frac{\ln\frac{H}{V}+\left(r_d-\frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_F\sqrt{T-t}}\right). \quad (2.6e)$$

(v) It is quite tricky to prescribe the far field boundary condition at $S \to \infty$. Instead of adopting an artificial boundary condition, we use the skew discretization technique where the discretization of the governing equation along the boundary nodes involves lattice nodes which are completely inside the computational domain.

Full two-way settlement

Under the full two-way settlement clause, B has to honor the swap contract even when A becomes default. The auxiliary conditions which model the payments paid by B upon the default of A have to be modified as follows:

(i) At maturity,

$$V(S,F,T) = \begin{cases} P_f S - P_d & \text{F>H} \\ P_f S - P_d & \text{F = H and } P_f S - P_d \leq 0 \\ (1-w)(P_f S - P_d) & \text{F=H and } P_f S - P_d > 0 \end{cases}. \quad (2.7a)$$

(ii) When the firm value F hits H,

$$V(S,H,t) = \begin{cases} (1-w)v(S,t) & \text{v(S, t)>0} \\ v(S,t) & \text{v(S, t)} \leq 0 \end{cases}. \quad (2.7b)$$

(iii) When the exchange rate S drops to zero,

$$V(0,F,t) = -P_d\left\{e^{-r_d(T-t)} + \frac{c_d}{r_d}[1-e^{-r_d(T-t)}]\right\}. \quad (2.7c)$$

Once the governing equations for the currency swap models are well formulated with full prescription of the auxiliary conditions, the equations can be solved numerically by standard finite difference method.

3. CHARACTERIZATION OF SWAP RATE SPREADS

We would like to analyze the impact on the swap rate spread under the influence of (i) firm credit rating, (ii) volatility of the firm value (iii) volatility of the exchange rate, (iv) correlation between firm value and exchange rate. Here, the swap spread is defined as the difference of the foreign swap rates c_f with and without the default possibilities of party A (recall that party B is assumed to be default free). Also, we examine the effects of the limited and full two-way settlement clauses on the swap spreads.

Recall that the foreign swap rate c_f gives the rate of continuous foreign payments from A to B. Suppose a settlement clause is more (less) favorable to party B, we would expect a decrease (increase) in c_f. Equivalently, this leads to negative (positive) swap spread on c_f in order that the swap contract is fair to both parties.

Firstly, we investigate the relationship between the swap spread on c_f and the credit rating of the defaultable party. Figure 2 shows the plots of swap spreads on c_f against F/H. For the given set of parameter values chosen in the swap model, the swap contract is in-the-money to the defaultable party A. It is observed that the swap spreads are negative for the limited two-way payment clause. This is expected as the limited settlement clause gives advantage to party B since B can be excused from honoring an out-of-the-money swap contract to itself when A defaults. The swap spread narrows as the credit rating of A improves, and becomes essentially zero as the ratio F/H goes beyond 3. For the full two-way settlement, the swap spread curve reveals that the swap spreads are always positive and the spread achieves a maximum value at certain level of F/H. The positivity of the swap spreads reflects the replacement cost to the non-defaulting party since it receives only a fraction of the market quotation of the swap contract upon default of the counterparty. Since the swap contract is in-the-money to the defaulting party A, the loss to counterparty B is zero when A defaults. Hence, the swap spread value becomes zero when F hits H. This explains why the swap spread curve corresponding to full two-way settlement increases from zero to some maximum value then decreases asymptotically to zero value at high firm value.

Secondly, we examine the relationship between the swap spread on c_f and the volatility of the firm value of A, σ_F (see Figure 3). When the firm value becomes more volatile, the possibility of default increases and the swap spreads become widened, that is, more negative for the limited two-way payment and more positive for the full two-way payment. The swap spreads tend to some asymptotic values at high σ_F.

Thirdly, the relationship between the swap spread and the volatility of the exchange rate is revealed in Figure 4. With the correlation coefficient between the firm value and exchange rate being positive, the firm has a higher possibility to default when the exchange rate becomes more volatile. Therefore, the value of swap spread increases in magnitude at higher value of σ_F for both full and limited two-way settlement clauses.

Lastly, the correlation coefficient ρ_{SF} is shown to have significant impact on the swap spreads (see Figure 5). For both full and limited two-way settlement clauses, the swap spreads are decreasing functions of the correlation coefficient. For full two-way settlement, the swap spread is always positive since B always loses upon the default of the counterparty A. Suppose the correlation is highly positive and S increases, there is a higher tendency for the increase of F (that is, less susceptible to default) so the expected loss to B associated with the replacement cost drops. On the other hand, when S decreases, the swap becomes more in-the-money to A. Correspondingly, the expected replacement cost incurs to B when A becomes default tends to a small value. Both arguments explain why the swap spread decreases when the correlation coefficient increases. For limited two-way settlement, the above argument of drop in swap spread with more positive correlation still applies, except that the swap spread can decrease beyond the zero value at high positive correlation.

4. CONCLUSIONS

In this paper, the contingent claim approach is employed to analyze the impact on the swap spreads of a currency swap with default risk of one of the counterparties. This paper goes beyond previous similar works by allowing intertemporal default of the defaultable party. It is observed that the limited and full two-way settlement clauses have significant effects on the behaviors of the swap spreads. The swap spreads depend sensibly on the proximity of the firm value to the defaulting threshold value, the volatilities of the firm value and exchange rate, and the correlation coefficient between firm value and exchange rate. It is observed that the full two-way settlement clause is always unfavorable to the non-defaulting party B so this leads to positive swap spread of the foreign swap rate (that is, defaultable party A should pay more in continuous foreign payments as compensation). The situation becomes more complicated for the limited two-way settlement clause. The non-defaulting party B may face losses (replacement cost) upon the default of the counterparty A. On the other hand, B is not required to honor the swap payment to A upon the default of A. Hence, the swap spread corresponding to the limited two-way settlement clause can be either positive or negative, depending on the relative strengths of the above two competing factors.

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FIGURES

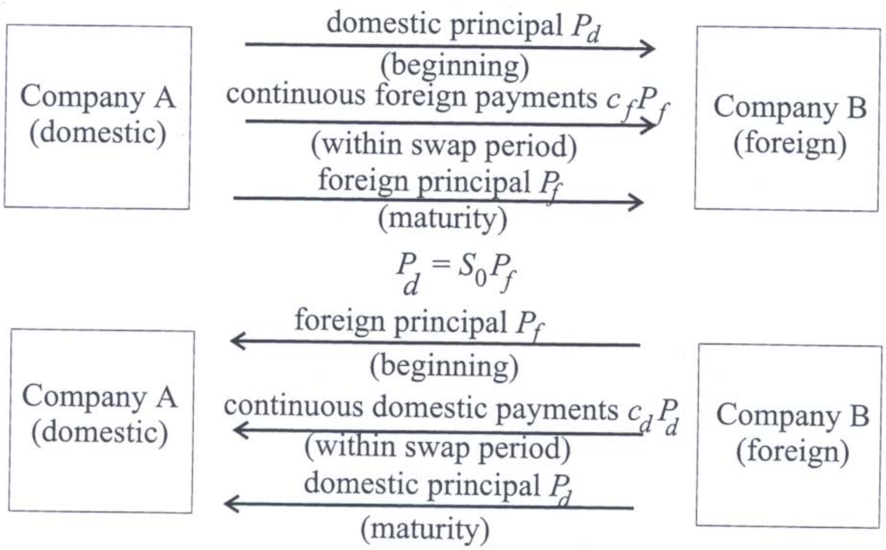

FIGURE 1. Cashflows between the two currency swap counterparties, assuming no intertemporal default.

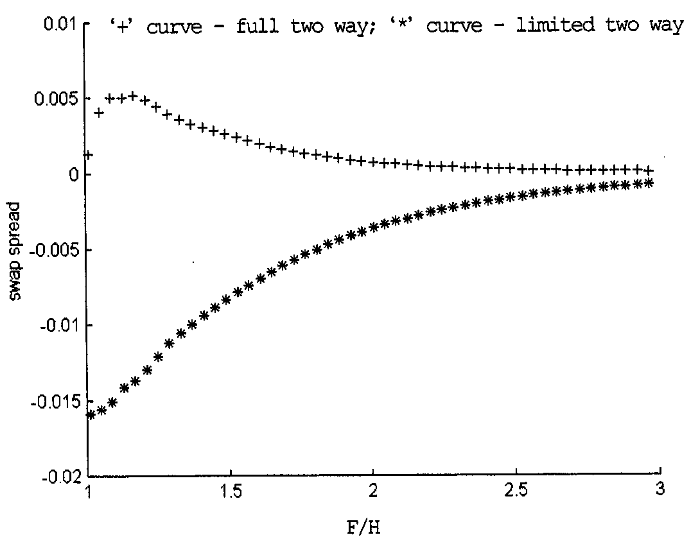

FIGURE 2. The relationship between the swap spread on c_f and the firm credit rating as measured by F/H, where F is the firm value and H is the default level. The parameter values are $P_d = 1, S_0 = 2, S = 2, c_d = 8\%, T = 4, r_d = r_f = 6\%, \sigma_S = 15\%, \sigma_F = 25\%, w = 0.75$ and $\rho_{SF} = 0.25$.

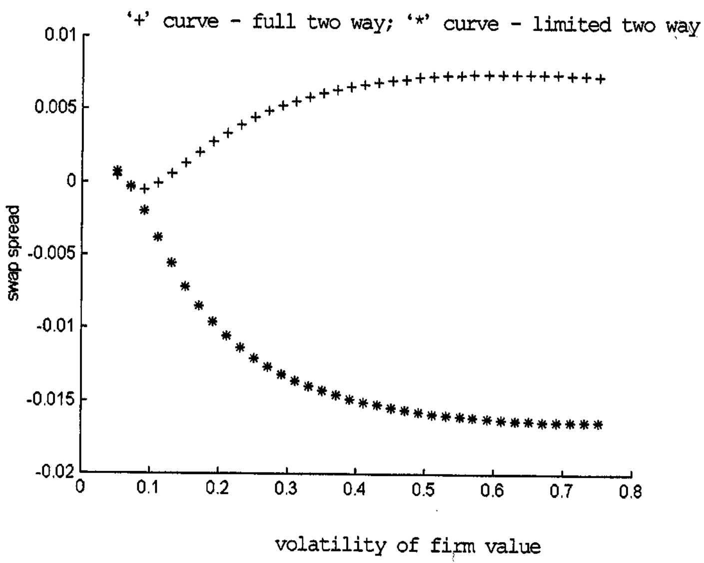

FIGURE 3. The relationship between the swap spread on c_f and the volatility of the firm value, σ_F. The same set of parameter values are used as in Figure 2, except that σ_F is allowed to vary and $H = 100, F = 125$.

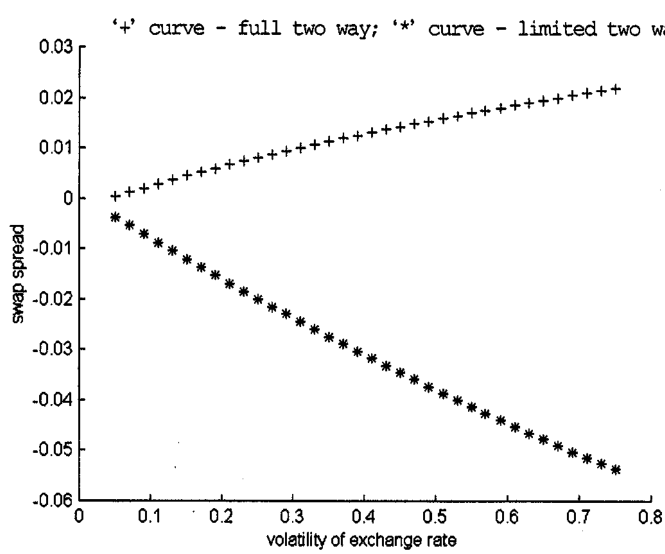

FIGURE 4. The relationship between the swap spread on c_f and the volatility of the exchange rate, σ_S. The same set of parameter values are used as in Figure 3, except that $\sigma_F = 25\%$ and σ_S is allowed to vary.

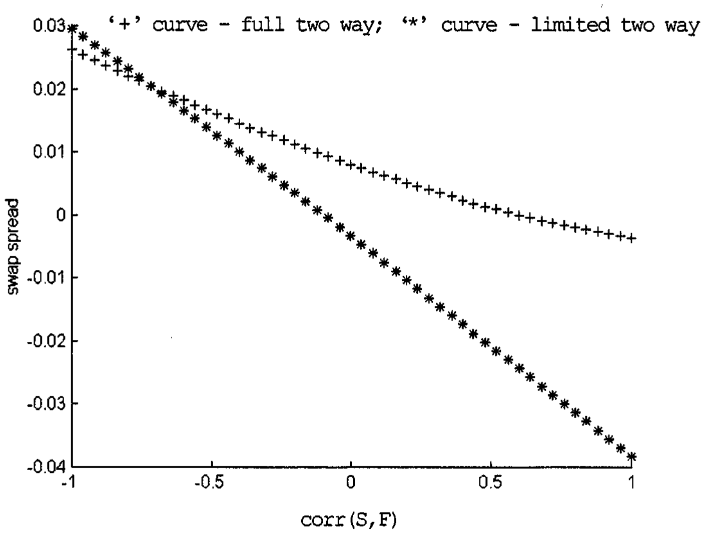

FIGURE 5. The relationship between the swap spread on c_f and the correlation coefficient between exchange rate and firm value, ρ_{SF}. The same set of parameter values are used as in Figures 3 and 4, except that ρ_{SF} is allowed to vary.