Mathematical modeling of default correlation of risky assets in a portfolio

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Defaults of a portfolio of risky obligors

$t = 0$

$t = 1$

$t = 2$

$t = 3$

same industrial sector – clusters phenomenon

Default means non-compliance of contract specification. The risky obligors may be corporate loan borrowers or bond issuers.
Default correlation – occurrence of multiple defaults in a portfolio of risky assets

*Sources of dependence between defaults*

- common macro-economic factors and sectors
  - correlation of the individual credit quality process to the economic cycle and sector indices

- default contagion – once a firm defaults, it may bring down
  (i) direct economic links between firms
  (ii) information effect e.g. accounting scandal of WorldCom
Basket default notes

- In the event that any of the loans defaults, the note is terminated. The bank keeps the $10 million of note proceeds, and the defaulted loan is put to the investor.
Any default correlation model should reflect

- Counterparty default:
  Default of one firm may trigger the default of other related firms.

- Defaults triggered by common factors:
  Default times tend to concentrate in certain periods of time (clusters of default).

Pricing considerations

- default intensities of obligors in the portfolio
- recovery rates upon default
- default correlation among the obligors
A good portfolio credit risk model should have the following properties

- Default dependence – produce default correlations of a realistic magnitude.

- Estimation – number of parameters should be limited.

- Timing risk – producing “clusters” of defaults in time, several defaults that occur close to each other

- Calibration (i) Individual term structures of default probabilities
  (ii) Joint defaults and correlation information

- Implementation - existence of a viable implementation mechanism, say Monte Carlo simulation method
(i) Infectious defaults (Davis and Lo, 2001)

Once a firm defaults, it may bring down the other firms with it.

Let $Z_i$ be the default indicator of Firm $i$ and $X_i$ be the “direct” default indicator of Firm $i$;

$Y_{ij}$ be an “infection” variable, which equals 1 when the default of Firm $i$ triggers the default of Firm $j$.

$$Z_i = X_i + (1 - X_i) \left[ 1 - \prod_{j \neq i} (1 - X_j Y_{ji}) \right]$$

$X_i, Z_i$ and $Y_{i,j}$ are Bernoulli random variables.
(ii) Propensity model

Upward jumps in the default intensity of non-defaulted firms at the default time of one of the default-correlated firms and/or the arrival of new information.

Default contagion with interacting intensities

Consider a portfolio of $N$ firms, a random default time $\tau_i$ is associated with each firm. Define $H^i_t = 1_{\{\tau_i \leq t\}}$. The default status of the assets in the portfolio is given by the default indicator process

$$H_t = (H^1_t, H^2_t, \ldots, H^N_t) \in \{0, 1\}^N = S,$$

where $S$ is the state space of $H_t$. Here, $H_t$ is modeled as a finite state Markov chain.
Markov chain formulation

The macroeconomic variables are described by a $d$-dimensional stochastic process

$$\Psi = (\psi_t)_{t \in [0, T]}.$$  

The overall state of the system is

$$\Gamma_t = (\Psi_t, H_t).$$

The information available to the investor in the market at time $t$ include the history of macroeconomic variables and default status of the portfolio up to time $t$.

$$\mathcal{F}_t = \mathcal{F}_t^\Psi \vee \mathcal{F}_t^1 \vee \mathcal{F}_t^2 \vee \cdots \vee \mathcal{F}_t^N$$

where the filtrations are generated collectively by the information contained in the state variables and the default processes, that is,

$$\mathcal{F}_t^\Psi = \sigma(\Psi_s : 0 \leq s \leq t)$$

$$\mathcal{F}_t^i = \sigma(H_s^i : 0 \leq s \leq t), \quad i = 1, 2, \ldots, N.$$
Write $\lambda_i(\psi_t, H_t)$ as the martingale default intensity of Firm $i$, that is

$$H_t^i - \int_0^{t \wedge \tau_i} \lambda_i(\psi_s, H_s) \, ds$$

is a $\mathcal{F}_t$-martingale.

Jarrow and Yu (2001) characterize the default intensity of firm $i$ by

$$\lambda_i(H_t) = \lambda_{ii} + \sum_{k=1}^{N} \lambda_{ik} \mathbf{1}_{\{\tau_k \leq t\}},$$

where $\lambda_{ik}$ are constants. The distributions of the default times are defined recursively through each other – looping default phenomenon.

The impact of defaults on the default intensities of surviving firms is exogenously specified, while the joint distribution of default times is then endogenously derived.
Let $y \in S$, $y$ is a vector of default indicators. Define the flipped state $\tilde{y}^i \in S$ by

$$\tilde{y}^i = 1 - y(i), \quad \tilde{y}^i(j) = y(j), \quad j \neq i.$$  

For $y_i, y_j \in S, y_j = \tilde{y}_i^k$ for some $k$ means $y_j$ is obtained by flipping the $k^{th}$ component in $y_i$.

Conditional on the given trajectory of $\Psi$, the process $H$ is a time-inhomogeneous finite state Markov chain. For $y_i, y_j \in S$, the infinitesimal generator $\Lambda_{[\psi]}(t) = (\Lambda_{ij}(t|\psi))_{|S| \times |S|}$ for $H$ at time $t$ given $\Psi_t = \psi$ is obtained by finding the transition rate $\Lambda_{ij}(t|\psi)$:
(a) for $i \neq j$

$$\Lambda_{ij}(t|\psi) = \begin{cases} 
[1 - y_i(k)]\lambda_k(\psi, y_i), & \text{if } y_j = \tilde{y}_i^k \text{ for some } k \\
0, & \text{else}
\end{cases}; \quad (1a)$$

(b) for $i = j$

$$\Lambda_{ii}(t|\psi) = -\sum_{j \neq i} \Lambda_{ij}(t|\psi) = -\sum_{k=1}^{N} [1 - y_i(k)]\lambda_k(\psi, y_i). \quad (1b)$$

For $i \neq j$, the transition rate $\Lambda_{ij}$ equals $\lambda_k(\psi, y_i)$ when $y_j$ can be obtained from $y_i$ by flipping its $k^{th}$ element from 0 to 1. The factor $1 - y_i(k)$ is added since $y_i(k) = 1$ is an absorbing state. Under this scenario, $\Lambda$ is seen to be upper triangular.
Define the conditional transition density matrix on $\psi_s = \psi'$ as

$$P(t, s|\psi') = (p_{ij}(t, s, y_i, y_j|\psi'))_{|S| \times |S|}.$$

**Backward Kolmogorov equation**

$$\frac{dP(t, s|\psi')}{dt} = -\Lambda_{[\psi']}(t)P(t, s|\psi'), \quad P(s, s|\psi') = I, \quad 0 \leq t \leq s.$$

The $(i, j)^{th}$ entry $p_{ij}(t, s|\psi')$ satisfies the following system of ODE:

$$\begin{cases}
\frac{dp_{ij}(t, s, y_i, y_j|\psi')}{dt} = -\sum_{k=1}^{|S|} \Lambda_{ik}(t|\psi)p_{kj}(t, s, y_k, y_j|\psi') \\
p_{ij}(s, s, y_i, y_j|\psi') = 1_{\{y_j\}}(y_i)
\end{cases}. \quad (2a)$$
Using the results in eqs (1a,b), eq. (2a) can be expressed as

\[
\frac{dp_{ij}(t, s, y_i, y_j|\psi')}{dt} + \sum_{k=1}^{N} [1 - y_i(k)] \lambda_k(\psi, y_i) [p_{ij}(t, s, \tilde{y}_i^k, y_j|\psi') - p_{ij}(t, s, y_i, y_j|\psi')] = 0
\]

with terminal condition:

\[
p_{ij}(s, s, y_i, y_j|\psi') = 1_{\{y_j\}}(y_i)
\] (2b)
**Forward Kolmogorov equation**

\[
\frac{dP(t, s|\psi')}{ds} = P(t, s|\psi') \Lambda[\psi'](s), \quad P(t, t|\psi') = I. \quad (3a)
\]

In a similar manner, we obtain

\[
\frac{dp_{ij}(t, s, y_i, y_j|\psi')}{ds} = \sum_{k=1}^{N} y(k) \lambda_{k}(\psi', y_j^k)p_{ij}(t, s, y_i, y_j^k|\psi')
- \sum_{k=1}^{N} [1 - y(k)] \lambda_{k}(\psi', y_j)p_{ij}(t, s, y_i, y_j|\psi'),
\]

with initial condition:

\[
 p_{ij}(t, t, y_i, y_j|\psi') = 1_{\{y_i\}}(y_j). \quad (3b)
\]
Marginal distribution of the default time

The distribution function is defined as

\[ F_i(t_i) = P_r(\tau_i \leq t_i) \quad \text{for} \quad i = 1, 2, \cdots, N. \]

We have

\[ F_i(t_i) = \int_D \sum_{y_j(i)=1} p_{1j}(0, t_i | \psi') dG_{\Psi_s}(\psi') \]

where \( G_{\Psi_s}(\cdot) \) is the distribution of \( \Psi_s \).

The first state in the Markov chain corresponds to “no default” for all obligors. Over \((0, t_i)\), we sum the Markov chain state which correspond to the occurrence of the default of Firm \( i \).
Joint distribution of the default times

The joint distribution of the default times is defined as

$$F(t_1, t_2, \cdots, t_N) = Pr(\tau_1 \leq t_1, \cdots, \tau_N \leq t_N).$$

Consider the case $t_1 \leq t_2 \leq \cdots \leq t_N$, we define

$$S(n, m) = \left\{ y \in S : n \leq \sum_{i=1}^{N} y(i) \leq m \right\}, \quad n \leq m.$$

Here, $S(n, m)$ is the set of default states with total number of defaults lying between $n$ and $m$, inclusively. Write $M = 2^N$, where $M$ is the total number of possible states. We take $y_1(k) = 0$ and $y_M(k) = 1$ for all $k = 1, 2, \cdots, N$. 
Under the time-homogeneous case, the distribution function can be expressed as

\[
F(t_1, t_2, \cdots, t_N) = \int_D \left[ p_{1M}(0, t_1|\psi') + \sum_{y_{j_1} \in S(1,N-1)} p_{1j_1}(0, t_1|\psi') p_{j_1M}(t_1, t_2|\psi) \\
+ \sum_{\substack{y_{j_1} \in S(1,N-1) \\
y_{j_2} \in S(2,N-1)}} p_{1j_1}(0, t_1|\psi') p_{j_1j_2}(t_1, t_2|\psi') p_{j_2M}(t_2, t_3|\psi') \\
+ \sum_{\substack{y_{j_1} \in S(1,N-1) \\
y_{j_2} \in S(2,N-1) \\
y_{j_N-1} \in S(N-1,N-1)}} p_{1j_1}(0, t_1|\psi') p_{j_1j_2}(t_1, t_2|\psi') \cdots p_{j_{N-1}M}(t_{N-1}, t_N|\psi') \right] dG_{\Psi_s}(\psi').
\]
Three-Firm Model

The inter-dependent default intensities of the 3 firms are defined as

\[
\begin{align*}
\lambda_t^A &= a_{10} + a_{12} \mathbb{1}_{\{\tau_B \leq t\}} + a_{13} \mathbb{1}_{\{\tau_C \leq t\}} + a_{14} \mathbb{1}_{\{\tau_B \leq t, \tau_C \leq t\}} \\
\lambda_t^B &= a_{20} + a_{21} \mathbb{1}_{\{\tau_A \leq t\}} + a_{23} \mathbb{1}_{\{\tau_C \leq t\}} + a_{24} \mathbb{1}_{\{\tau_A \leq t, \tau_C \leq t\}} \\
\lambda_t^C &= a_{30} + a_{31} \mathbb{1}_{\{\tau_A \leq t\}} + a_{32} \mathbb{1}_{\{\tau_B \leq t\}} + a_{34} \mathbb{1}_{\{\tau_A \leq t, \tau_B \leq t\}}.
\end{align*}
\]

We assume an extra jump in default intensity if the other two firms have defaulted, allowing the interaction between the default events on the intensity of surviving firms.

The state space $S$ of $H = (H_t^A, H_t^B, H_t^C)$ is given by

\[
S = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}.
\]

<table>
<thead>
<tr>
<th>State 1</th>
<th>(0,0,0)</th>
<th>State 2</th>
<th>(1,0,0)</th>
<th>State 3</th>
<th>(0,1,0)</th>
<th>State 4</th>
<th>(0,0,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 5</td>
<td>(1,1,0)</td>
<td>State 6</td>
<td>(1,0,1)</td>
<td>State 7</td>
<td>(0,1,1)</td>
<td>State 8</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>
The infinitesimal generator $\Lambda$ of the process $H$ is given by

\[
\Lambda = \begin{bmatrix}
-a_{10} - a_{20} - a_{30} & a_{10} & a_{20} & a_{30} & 0 & 0 & 0 \\
0 & -a_{20} - a_{21} & -a_{20} - a_{21} & 0 & a_{20} + a_{21} & a_{30} + a_{31} & 0 \\
0 & 0 & -a_{10} - a_{12} & a_{10} + a_{12} & 0 & a_{30} + a_{32} & 0 \\
0 & 0 & 0 & a_{10} + a_{13} & a_{10} + a_{13} & a_{20} + a_{23} & 0 \\
0 & 0 & 0 & 0 & -(a_{30} + a_{31} + a_{32} + a_{33}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -(a_{20} + a_{21} + a_{22} + a_{23}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(a_{10} + a_{12} + a_{13} + a_{14}) \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

For example, consider the transition rate from State 2: $(1 \ 0 \ 0)$ to State 5: $(1 \ 1 \ 0)$ and State 6: $(1 \ 0 \ 1)$:

\[
\Lambda_{25} = a_{20} + a_{21}
\]
\[
\Lambda_{26} = a_{30} + a_{31}
\]
\[
\Lambda_{21} = \Lambda_{23} = \Lambda_{24} = \Lambda_{27} = \Lambda_{28} = 0 \text{ and } \Lambda_{22} = 1 - \Lambda_{25} - \Lambda_{26}.
\]
The transition density matrix $P(t)$ can be obtained by solving the forward Kolmogorov equation.

\[
\frac{dP(t)}{dt} = P(t) \Lambda, \quad P(0) = I.
\]

The distribution functions of the default times are found to be

\[
F_A(t) = P_r(\tau_A \leq t) = P_{12}(t) + P_{15}(t) + P_{16}(t) + P_{18}(t)
\]
\[
F_B(t) = P_r(\tau_B \leq t) = P_{13}(t) + P_{15}(t) + P_{17}(t) + P_{18}(t)
\]
\[
F_C(t) = P_r(\tau_C \leq t) = P_{14}(t) + P_{16}(t) + P_{17}(t) + P_{18}(t)
\]

The joint distribution of default times $\tau_A, \tau_B$ and $\tau_C$ is defined as

\[
F(t_1, t_2, t_3) = P_r(\tau_A \leq t_1, \tau_B \leq t_2, \tau_C \leq t_3).
\]
The distribution function takes different form under various scenarios of relative magnitudes of \( t_1, t_2 \) and \( t_3 \). For example, suppose \( t_1 \leq t_2 \leq t_3 \), then

\[
F(t_1, t_2, t_3) = P_{18}(t_1) + P_{12}(t_1)P_{28}(t_2 - t_1) + P_{15}(t_1)P_{58}(t_3 - t_2) \\
+ P_{16}(t_1)P_{68}(t_2 - t_1) + P_{12}(t_1)P_{25}(t_2 - t_1)P_{58}(t_3 - t_2) \\
+ P_{15}(t_1)P_{55}(t_2 - t_1)P_{58}(t_3 - t_2).
\]
Joint density function of the default times

\[ f(t_1, t_2, t_3) = \begin{cases} 
  a_{10}(a_{20} + a_{21})(a_{30} + a_{31} + a_{32} + a_{34}) \\
  e^{-(a_{10} - a_{21} - a_{31})t_1 - (a_{20} + a_{21} - a_{32} - a_{34})t_2 - (a_{30} + a_{31} + a_{32} + a_{34})t_3} & t_1 \leq t_2 \leq t_3 \\
  a_{10}(a_{30} + a_{31})(a_{20} + a_{21} + a_{23} + a_{24}) \\
  e^{-(a_{10} - a_{31} - a_{21})t_1 - (a_{30} + a_{31} - a_{23} - a_{24})t_2 - (a_{20} + a_{21} + a_{23} + a_{24})t_2} & t_1 \leq t_3 \leq t_2 \\
  a_{20}(a_{10} + a_{12})(a_{30} + a_{31} + a_{32} + a_{34}) \\
  e^{-(a_{20} - a_{12} - a_{32})t_2 - (a_{10} + a_{12} - a_{31} - a_{34})t_1 - (a_{30} + a_{31} + a_{32} + a_{34})t_3} & t_2 \leq t_1 \leq t_3 \\
  a_{30}(a_{10} + a_{13})(a_{20} + a_{21} + a_{23} + a_{24}) \\
  e^{-(a_{30} - a_{13} - a_{23})t_3 - (a_{10} + a_{13} - a_{21} - a_{24})t_1 - (a_{20} + a_{21} + a_{23} + a_{24})t_2} & t_3 \leq t_1 \leq t_2 \\
  a_{20}(a_{30} + a_{32})(a_{10} + a_{12} + a_{13} + a_{14}) \\
  e^{-(a_{20} - a_{12} - a_{32})t_2 - (a_{30} + a_{32} - a_{13} - a_{14})t_3 - (a_{10} + a_{12} + a_{13} + a_{14})t_1} & t_2 \leq t_3 \leq t_1 \\
  a_{30}(a_{20} + a_{23})(a_{10} + a_{12} + a_{13} + a_{14}) \\
  e^{-(a_{30} - a_{13} - a_{23})t_3 - (a_{20} + a_{23} - a_{12} - a_{14})t_2 - (a_{10} + a_{12} + a_{13} + a_{14})t_1} & t_3 \leq t_2 \leq t_1 
\end{cases} \]
Contagion risk based on updating of beliefs

Default intensity of Firm $i$

$$\lambda_i(t) = \begin{cases} 
\lambda_i^H & \text{if the economy is in the state } H \\
\lambda_i^L & \text{if the economy is in the state } L 
\end{cases}.$$ 

Investors do not know whether the economy is in state $H$ or $L$, but form a prior $p^H(t) = P[H|F_t]$, where $F_t$ represents all information investors have available at date $t$.

$$\bar{\lambda}_i(t) = p^H(t)\lambda_i^H + [1 - p^H(t)]\lambda_i^L,$$

where $\bar{\lambda}_i(t)$ is defined as

$$E_t \left[ d\mathbf{1}_{\{\tau_i < t\}} \right] = \bar{\lambda}_i(t)\mathbf{1}_{\{\tau_i > t\}} dt.$$
Investors continuously update their estimate of $p^H(t)$ conditional upon whether or not they observe a default event during $dt$.

$$\frac{dp^H(t)}{p^H(t)} = \sum_{i=1}^{N} \left[ \left( \frac{\lambda_i^H}{\lambda_i(t)} - 1 \right) \left( d1_{\{\tau_i \leq t\}} - \lambda_i(t) 1_{\{\tau_i > t\}} \right) dt \right]$$

- If the prior $p^H$ is either 0 or 1, then there is no updating.
- When no default is observed over an interval $dt$, investors revise downward the probability of being in the high-default state.
- When a default is observed, they revise upward the probability that the economy is in the high default-state.
Counterparty risk of credit default swap

3 parties: Protection Seller, Protection Buyer, Reference Obligor
How does the inter-dependent default risk structure between the Protection Seller and the Reference Obligor affect the swap rate?

1. *Replacement cost* (Seller defaults earlier)

   - If the Protection Seller defaults prior to the Reference Entity, then the Protection Buyer renews the CDS with a new counterparty.
   - Supposing that the default risks of the Protection Seller and Reference Entity are positively correlated, then there will be an increase in the swap rate of the new CDS.
2. *Settlement risk* (Reference Entity defaults earlier)

- The Protection Seller defaults during the settlement period after the default of the Reference Entity.
The inter-dependent default risk structure between Protection Seller $B$ and Reference Entity $C$ is characterized by the correlated default intensities:

$$
\lambda_t^B = b_0 + b_2 \mathbf{1}_{\{\tau_C \leq t\}}
$$

$$
\lambda_t^C = c_0 + c_2 \mathbf{1}_{\{\tau_B \leq t\}}.
$$

The joint density of default time $(\tau_B, \tau_C)$ is

$$
f(t_1, t_2) = \begin{cases} 
    c_0 (b_0 + b_2) e^{-(b_0 + b_2) t_1} - (c_0 - b_2) t_2, & t_2 \leq t_1, \\
    b_0 (c_0 + c_2) e^{-(c_0 + c_2) t_2} - (b_0 - c_2) t_1, & t_2 > t_1.
\end{cases}
$$
Buyer pays $S(T)$ at $T_i$, provided $\tau^B \wedge \tau^C > T_i$, $i = 1, 2, \cdots, n$.

We compute the swap rate $S(T)$ by setting

\[
\text{expected present value of Protection Buyer payment} = \text{expected present value of compensation payment at } \tau^C + \delta, \text{ with } \tau^C < T \text{ and } \tau^B > \tau^C + \delta,
\]

where $\delta$ is the settlement period.
The swap rate $S(T)$ under this two-firm model is determined by

$$
\sum_{i=1}^{n} E[e^{-rT_i} S(T) 1_{\{\tau_B \land \tau_C > T_i\}}] + S(T) A(T)
$$

$$
= E \left[ e^{-r(\tau^C + \delta)} 1_{\{\tau_C \leq T\}} 1_{\{\tau_B > \tau_C + \delta\}} \right],
$$

where $S(T) A(T)$ is the present value of the accrued swap premium for the fraction of period between $\tau^C$ and the last payment date.

$$
A(T) = \sum_{i=1}^{n} E \left[ e^{-r\tau_C} \left( \frac{\tau_C^C - T_{i-1}}{\Delta T} \right) 1_{\{T_{i-1} < \tau_C < T_i\}} 1_{\{\tau_B > \tau_C\}} \right].
$$
Settlement risk

Suppose the Protection Seller is default-free, the swap premium is then given by

$$\sum_{i=1}^{n} E[e^{\tau_{C}T_{i}}\bar{S}(T)1_{\{\tau_{C}>T_{i}\}}] + \bar{S}(T)\bar{A}(T) = E\left[e^{-r(\tau_{C}+\delta)}1_{\{\tau_{C}\leq T\}}\right],$$

where

$$\bar{A}(T) = \sum_{i=1}^{n} E\left[e^{-r\tau_{C}}\left(\frac{\tau_{C}-T_{i}-1}{\Delta T}\right)1_{\{T_{i-1}<\tau_{C}<T_{i}\}}\right].$$

To examine the effect of settlement risk on the swap premium, we define the swap premium spread as the difference of the swap premium with and without settlement risk.
Dependence of settlement risk premium on $\delta$. The base parameter values are: $r = 0.05$, $\Delta T = 0.25$, $b_0 = 0.15$, $b_2 = 0.15$, $c_0 = 0.1$, $c_2 = 0.1$. 
Dependence of settlement risk premium on $\delta$

Seller: $\lambda^B_t = b_0 + b_2 \mathbb{1}_{\{\tau^C < t\}}$

Reference Entity: $\lambda^C_t = c_0 + c_2 \mathbb{1}_{\{\tau^B < t\}}$

- The underlying intensity value of $\lambda^C_t$, $c_0$, has the strongest influence on the settlement risk premium.

- The intensity values of $\lambda^B_t$ have less influence.

- The default correlation between $B$ and $C$, proxied by $b_2$, is slightly more important than $b_0$. 
Without default risk of the Protection Seller $B$, Protection Buyer faces

(i) paying higher expected present value of total swap payments to the Seller since

$$
E \left[ e^{-rT_i} \mathbb{1}_{\{\tau_C > T_i\}} \right] > E \left[ e^{-rT_i} \mathbb{1}_{\tau_B \land \tau_C > T_i} \right];
$$

(ii) receiving higher expected present value of contingent payment from the Seller

$$
E \left[ e^{-r(\tau_C + \delta)} \mathbb{1}_{\{\tau_C \leq T\}} \right] \geq E \left[ e^{-r(\tau_C + \delta)} \mathbb{1}_{\{\tau_C \leq T\}} \mathbb{1}_{\{\tau_B > \tau_C + \delta\}} \right],
$$

since there is no settlement risk.