Homework No.3 for Math 3121

Due Date: Oct 25.

Announcement: the **quiz** will be held on Oct. 24 (Tuesday), 9:00-10:20am at the classroom 2302.

Problem 1. Let $\sigma \in S_8$ be of the form

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 3 & 2 & 1 & 5 & 8 & 7 \end{pmatrix}.$$

(1) Computer σ^2. (2). Decompose σ as a product of disjoint cycles. (3). Compute the order of σ. (4). Compute σ^{-1}.

Problem 2. Let $\sigma \in S_8$ be of the form

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 3 & 2 & a & b & 1 & 7 \end{pmatrix}.$$

Suppose σ is an odd permutation,
(1). Find a and b. (2). Decompose σ as a product of disjoint cycles.
(3). Compute the order of σ. (4). Decompose σ^{-1} as a product of disjoint cycles. (5). Comupte σ^{2017}.

Problem 3. Which of the following is a coset of the subgroup $H = \{e, (12)\}$ in S_3?
(1). $B_1 = \{(123), (132), e\}$.
(2). $B_2 = \{(123), (12)\}$.
(3). $B_3 = \{(123), (13)\}$.
(4). $B_4 = \{(123), (132)\}$.
(5). $B_5 = \{e, (132)\}$.

Problem 4. Let G be an abelian group, prove that $H = \{a \in G \,|\, a^3 = e\}$ is a subgroup of G.