Homework No.3 for Math 3121

Due Time: Oct 17, 6pm.

Problem 1. Let $\sigma \in S_8$ be the element

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 3 & 2 & 1 & 5 & 8 & 7 \end{pmatrix}.$$

(1) Compute σ^2. (2). Decompose σ as a product of disjoint cycles. (3). Compute the order of σ. (4). Compute σ^{-1}.

Problem 2. Let $\sigma \in S_8$ be of the form

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 3 & 2 & a & b & 1 & 7 \end{pmatrix}.$$

Suppose σ is an **odd** permutation,

(1). Find a and b. (2). Decompose σ as a product of disjoint cycles.

(3). Compute the order of σ. (4). Decompose σ^{-1} as a product of disjoint cycles. (5). Comupte σ^{2019}.

Problem 3. Give an example of a subgroup in S_4 that has order 6.

Problem 4. Let σ and τ denote the transpositions (12) and (23) in S_8. Prove that $\sigma\tau\sigma = \tau\sigma\tau$.

Problem 5. Let G be an abelian group, prove that $H = \{a \in G \,|\, a^3 = e\}$ is a subgroup of G.