Homework No.4 for Math 3121

Deadline: Nov. 14, 6:00pm

Problem 1. Determine if the following maps are homomorphisms of groups (No reasons needed).

(1). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = 2019a$

(2). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = a^{2019}$

(3). $\Phi : \mathbb{R} \to \mathbb{R}, \quad \Phi(a) = 2019a$

(4). $\Phi : \mathbb{R} \to \mathbb{R}, \quad \Phi(a) = a^2$

(5). $\Phi : GL(n, \mathbb{R}) \to \mathbb{R}^*, \quad \Phi(A) = Det(A)^{10}$.

(6). $\Phi : \mathbb{R} \to \mathbb{R}^*, \quad \Phi(a) = 10^a$.

(7). $\Phi : \mathbb{R}^* \to \mathbb{R}, \quad \Phi(a) = 10^a$.

(8). $\Phi : \mathbb{R}^* \to \mathbb{R}, \quad \Phi(a) = \log_{10}(a^2)$.

(9). $\Phi : S_3 \to S_4$,

$$\Phi(\sigma) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ i & j & k & 4 \end{pmatrix}, \quad \text{where } \sigma = \begin{pmatrix} 1 & 2 & 3 \\ i & j & k \end{pmatrix}.$$

Problem 2. Find a homomorphism $\Phi : \mathbb{C}^* \to \mathbb{C}^*$ such that $Ker(\Phi) = U_5$ (no reasons needed).

Problem 3. Let A and B be groups. Find an isomorphism $\Phi : A \times B \to B \times A$.

Problem 4. Let G be a finite group, H_1 and H_2 be subgroups of G. (1). Prove that $H_1 \cap H_2$ is a subgroup of G. (2). If $|H_1|$ and $|H_2|$ are relatively prime, prove that $H_1 \cap H_2 = \{e\}$.

Problem 5. Let G and G' be finite groups, suppose that $|G|$ and $|G'|$ are relatively prime, prove that a homomorphism $\phi : G \to G'$ must be trivial, i.e., $\phi(a) = e'$ for all $a \in G$,