# GATEWAY TESTING IN MATHEMATICS-PAST AND PRESENT 

Robert Megginson
University of Michigan
Department of Mathematics

## The problem

- In introductory college mathematics, there are certain computations students should know how to do quickly and confidently.
- In this day of easy access to handheld computational aids and computer algebra systems, why? An example will be given.
- But you don't want students to mistake that for the main content material of the course.
- They should not believe that the ultimate goal of calculus is to be able to find the antiderivative of every rational combination of trigonometric functions.
- And they will conclude that if a significant portion of the final course grade comes from testing computational skills.


## An example involving algorithm efficiency

- Why make students do enough examples of the Euclidean algorithm to get a good feel for how it is working? After all, computers can find greatest common divisors of even big pairs of positive integers with a few push of the buttons on a personal computer.
- The number sense developed by doing so makes it transparently obvious why a step in the Euclidean algorithm involving two integers such that one is slightly more than half the other does not help you get to the final answer very fast.
- And then doing the computation for two fairly large consecutive terms in the Fibonacci sequence makes it transparently obvious why one is dealing with a worst-case scenario for requiring lots of iterations to find the g.c.d.of two numbers of a certain size.
- From there it is a fast trip to this important result:

Lamé's theorem. Suppose that $a, b$ are two positive integers such that $a<b$ and that $a<F_{n}$, the $n^{\text {th }}$ term in the Fibonacci sequence $\left(F_{0}, F_{1}, F_{2}, \ldots\right)=(1,1,2, \ldots)$. Then the Euclidean algorithm will find the g.c.d. of $a$ and $b$ in at most $n-3$ steps, which is less than 5 times the number of decimal digits in $a$.

Theorem (Division algorithm, special case). Let $F_{p}$ be the finite field with prime $p$ elements and let $f, g$ be nonzero polynomials with coefficients in $F_{p}$. Then there are unique polynomials $q, r$ with coefficients in $F_{p}$ such that $f=q g+r$ and the degree of $r$ is less than the degree of $g$.

- Straightforward proof: Dispose of the special case where the degree of $g$ is greater than the degree of $f$, then finish off by induction on the degree of $f$.
- Students will be able to check the proof to see that it works, and, if they know it could be required, regurgitate it on a test.
- Alternative approach: Have students practice polynomial long division in this setting with carefully chosen exercises, then give the proof and tie it back to the exercises.
- More complete understanding; also, they'll get an idea of why the procedure suggests that the proof will be an induction argument.


## An experiment conducted years ago by Patricia Shure in our department

- A standard question was asked on the final exam of an introductory calculus course: "For the given function $f$ and given starting value $x_{0}$, find the value $x_{1}$ resulting from one iteration of Newton's method."
- Almost everyone could do it.
- Second part to the question added by Pat (on a whim): "What is the purpose of Newton's method?"
- Answer given by almost every one of the thousand students:
"Starting with an $x_{0}$, use Newton's method to find an $x_{1}$."
- Bad teaching? Strong evidence that this is not the problem.
- Students treat as important knowledge what they have figured out will "be on the test", when that truly factors into the course grade.


## An alternative for regular testing:

- Reserve for main tests, where performance level influences final course grade, questions that test understanding.
- May require computational skills, but as tools clearly subservient to the main points of understanding to be tested.
- Example: "A hydrogen balloon launched at ground level at time $t_{0}=0$ with a positive initial velocity rose directly upward for some period of time with no acceleration, then broke so that it instantly lost all its gas, falling back to earth without bouncing after it struck. Sketch a graph of the balloon's vertical velocity $v$ as a function of time $t$ up to and past the time it struck the ground, where all important features of the graph's shape are clearly evident."


## Pure skill testing reserved for gateway tests

- Examples:
- "Find formulas for the derivatives of these ten functions."
- "Find the values of these five definite integrals."
- And, yes, "For the given function $f$ and given starting value $x_{0}$, find the value $x_{1}$ resulting from one iteration of Newton's method."
- Features of such tests
- "Closed book," with no technology aids allowed
- Enforced time limit
- No part credit given on any exercise
- High performance level expected
- But the test may be retaken (a different version each time) until passed, with a required interval between attempts and a deadline date
- No impact on the course grade if finally passed, but a significant reduction in the final grade if not passed; no exceptions


## One significant additional advantage of gateway testing

- Students will give up learning material they believe difficult or tedious, in anticipation that they can "make up the difference" in grade outcome by knowing material with which they are more comfortable (or think is easier) better.
- With gateway testing, you can't usually get away with that.
- An example from one student's exasperated comment on a gateway test on which the student kept missing a chain rule exercise: "HOW MANY TIMES DO I HAVE TO PROVE TO YOU THAT I CAN’T DO THIS!!!"
- Great diagnostic tool for instructors.


## Can also be used to reinforce prerequisite

 material- Example: Our Calculus II course has an introductory gateway test that forces students to review material from Calculus I (which they may not have taken at U-M)
- Students with particularly weak incoming skill sets can be counseled (cautiously) to retake Calculus I


## MY PERSONAL JOURNEY THROUGH GATEWAY TESTING

And the lessons learned along the way

## First used them in the early 1980s in calculus at Eastern Illinois University

- I DID NOT INVENT THE IDEA OR THE TERM! (There seems to be an urban legend out there that I did.)
- Called "barrier tests" by some, probably earlier than the 1980s
- My earliest use, in Calculus I:
- Five question derivative test + separate and later five question integral test, administered on paper
- Given in class weekly in different versions for three weeks or so, then weekly during my office hours
- Perfect score required to pass
- Deadline: Morning of the final exam
- Students would simply fail the course if either test was not passed
- Purpose: Force solid learning of basic skills; standard testing would not do that.


## LESSONS LEARNED

- Don't require absolute perfection. These tests became seven or eight question tests on which students were allowed to miss one. But it was still the case (and remains so today) that "small" errors were not forgiven.
- Have a deadline earlier than the last possible moment.
- But in those earlier days, with the extreme consequence of not passing, I also had an unadvertised way that students could get an extra attempt above and beyond that, but requiring some extra effort on their part.
- With fixed intervals between tests, students needed practice versions.


## On to the University of Michigan in 1992

- Hired as an Associate Professor of Mathematics
- Directed the U-M Mathematics Tutoring Laboratory, "Math Lab" (more on that tomorrow)
- Also was course director for our Precalculus Course, $\approx 1000$ students per year
- Instituted gateway testing in precalculus, using "shining new technology"


## University of Michigan precalculus gateway testing, 1992

- First attempt at computer generation, but still printing paper copies
- Five exercises, covering five basic skills important in the course
- Each exercise selected randomly from a database of 40 questions testing that skill; not multiple choice
- For "ease" of grading, exercises numbered 1-40 in database for first exercise, 41-80 in second database, ... , 161-200 in fifth database; sequence number printed in front of exercise
- "Program" doing the selecting: enormous Lotus 123 spreadsheet
- Printed in batches of 100


## 1992 administration

- Instructors gave first round of tests in class
- After round 1, students could take in proctored mode in Math Lab, but no more than once per day; at instructor discretion, tests could also be taken during office hours
- Instructors graded own students' tests
- With five questions, perfection again required


## LESSONS LEARNED

- Requiring perfection was no better an idea in 1992 than a decade earlier.
- Instructors would not enforce the perfection rule if students made one "silly" error.
- Instructors hated the grading; the use of a 200-question answer key did not make it any more pleasant.
- Most students were quite honest, but those who were not found interesting ways to be dishonest
- Having friends take the test for them; sometimes worked even with ID required
- Obtained blank copies of a test, did it ahead of time, and brought it in to the Math Lab
- Difficult for Math Lab monitors to proctor
- A few students obtained copies of the key, brought it into test
- Students collaborated to create listings of entire database, with solutions, worked all exercises but didn't bring listing to Math Lab

1993

- "New wave" reformed calculus implemented at Michigan
- Derivative gateway test put into place in Calculus I (differential calculus)
- Both precalculus and calculus tests had 7 exercises, BUT
- Students were allowed to miss one (so 6/7 passing; still true)


## LESSONS LEARNED

- Problem eliminated with students complaining about "pickiness" when one careless "typo"-type error caused a failure
- Problem eliminated (mostly) with instructors who would forgive one "small" error
Other lessons learned but not addressed still remained, plus several new ones:
- Some instructors not under my control did not buy into the system, and told their students so (and instructors still generally loathed the grading)
- Large number of students now taking the test overflowed the Math Lab


## Solution to instructors not buying in

- During instructor training, course directors repeatedly emphasize to instructors the reason for gateway testing; this is crucial
- Instructors are asked (required) to inform their students about the reasons for gateway testing


## Solution to instructors' grading pain

- Math Lab tutors took over all grading except first in-class gateway test and as instructors desired
- But instructors did expect one-day turnaround, and that could not always be provided.


## Definition of a Wicked Problem

- Definition by Horst Rittel and Melvin Webber (1973): Roughly, incompletely, and somewhat incorrectly, a wicked problem is one for which every attempt at a solution unveils another problematic aspect of the system


## Partial solution for slow turnaround

- I bit the bullet and converted all of the tests to multiple (5) choice format.
- I think this is a very bad idea for basic skill tests such as these.
- Aggravated the problem of students collecting lists of solutions, and the answer key did get loose.


## Solution for students not studying between attempts

- Though students were theoretically limited to one attempt per day, we decided that students could have a second if they were to sit down with a Math Lab tutor and go over all of their errors on the first attempt of the day, and demonstrate that they could do that type of exercise.
- Under the current system that still is the case, except that an instructor can insist that her or his students get help directly from the instructor.
- Personal suspicions about why that change was made.


## The good news and the bad news

- Good news
- Far fewer complaints from instructors and students
- Easier to administer and grade
- Bad news
- Observation: Despite exhortations to work out exercises on scratch paper before selecting an answer, students were generally just selecting answers without doing that
- Grading, though easier, still created backups as deadlines neared


# With the $21^{\text {st }}$ Century just around the corner, we decided it was time to enter it 

## Moved gateway tests to an online WeBWorK based system

- Obtained US\$200,000 funding from the U.S. National Science Foundation (NSF DUE 0088264) for implementation
- Dedicated a computer laboratory near the Math Lab tutoring center to testing
- Hired Gavin LaRose to study possible software platforms and do the coding; he selected WeBWorK and it was a great choice
- He has remained with us as a very valuable member of our faculty (at senior lecturer rank)
- Abandoned (to my great joy) multiple choice tests


## Practice gateway is available some time before the real test is put on line; here is what students first see:

## Hints and Tips:

- You login to the system through the University weblogin system. Therefore, your username is your uniqname, and your password the Kerberos password you use for your e-mail, etc.
- Take the gateway for practice until you can consistently pass it. You may practice the test as many times per day as you like.
- After passing the practice test, go to the proctored Gateway lab (B069 in East Hall) to take it for a grade.
- A passing score on the gateway is $6 / 7$ or better.
- The time limit for the gateway is 30 min .
- Only proctored tests count for passing the gateway.
- You may take up to two proctored tests per day. To take it a second time, you must go over a failing test with a tutor in the Math Lab.
- Books, notes, and other aids such as calculators are not permitted on the gateway.
- Always work problems out by hand, on paper, before entering them in the test.
- Watch Your Case: X is different from x , $\operatorname{Sin}$ is different from sin, and Pi is different from pi. Always use the same case as the problem you're solving.
- Always preview your answers.


## A demonstration

- Derivative gateway test for Calculus I


## 2013-14 Gateway Lab Usage

| Gateway <br> Test | Course <br> Enrollment | Visits for <br> Testing | Pass Rates |
| :--- | :---: | :---: | :---: |
| Precalculus | 568 | 2692 | $90 \%$ |
| Differential <br> Calculus | 2162 | 4893 | $98 \%$ |
| Integral <br> Calculus <br> (two tests) | 1480 | 7447 | Entrance: $99 \%$ <br> Integral: $93 \%$ |
| Multivariable <br> Calculus | 2110 | 4598 | $95 \%$ |
| Total | 6320 | 19630 |  |

Assessment [LaRose and Megginson, Implementation and assessment of on-line gateway testing, Primus 13 (December 2003), no. 4, 289-307]

- "We directly tested students' skills before and after they took a gateway test [which may have included multiple attempts], and determined that the scores after the gateway increased significantly from those before."
- What does that mean?
- Before taking the gateway test, the students did homework and should have been well enough prepared to do the exercises to deal with the standard way their skills would have been tested.
- After taking the gateway test, they could execute those skills significantly better.


## WeBWorK -based software availability

- Can be obtained from the Mathematical Association of America website (www.maa.org)
- But figuring out how to find it there can be a little confusing. For assistance, contact Gavin LaRose, glarose@umich.edu

