

Lecture Notes

Introduction to Cluster Algebra

Ivan C.H. Ip*

§0 Introduction

Cluster algebra was introduced in 2000 by Sergei Fomin and Andrei Zelevinsky. Motivation from Lie theory (semisimple algebraic groups):

- Total positivity
- Dual canonical basis

The combinatorial data used to define cluster algebra shows up in a lot of different areas, and sparks a lot of recent research (over 2000 papers!)

- Quiver representations
- Root systems
- Quantum groups
- Discrete integrable systems
- Combinators
- Poisson geometry
- Teichmüller theory

This course will serve as a very basic introduction to the theory of cluster algebra. We hope that by the end of the course, students will have enough working knowledge to study more literature related to cluster algebra.

This introductory course will be divided into 4 parts:

- (1) Examples and definitions
- (2) Properties and classification

*Center for the Promotion of Interdisciplinary Education and Research/
Department of Mathematics, Graduate School of Science, Kyoto University, Japan
Email: ivan.ip@math.kyoto-u.ac.jp

(3) Upper bounds and double Bruhat cells

(4) Applications

Although the definitions are standard from the very first publication by Fomin and Zelevinsky's "*Cluster Algebra I - IV*", the examples and presentation styles given in this lecture notes are mostly inspired from various courses and surveys which can be found in the "Cluster Algebras Portal" (<http://www.math.lsa.umich.edu/~fomin/cluster.html>).