Bonds and Yield to Maturity
**Bonds**

A bond is a debt instrument requiring the issuer to repay to the lender/investor the amount borrowed (par or face value) plus interest over a specified period of time.

Specify (i) maturity date when the principal is repaid; 
(ii) coupon payments over the life of the bond.
Cash flows in bonds

1. Coupon rate offered by the bond issuer represents the cost of raising capital (reflection of the creditworthiness of the bond issuer).

2. Assume the bond issuer does not default or redeem the bond prior to maturity date, an investor holding this bond until maturity is assured of a known cash flow pattern.
Other features in bond indenture

1. *Floating rate bond* – coupon rates are reset periodically according to some predetermined financial benchmark.


3. *Callable feature* (callable bonds)
   The issuer has the right to buy back the bond at a specified price. Usually this call price falls with time, and often there is an initial call protection period wherein the bond cannot be called.
4. **Put provision** – grants the bondholder the right to sell back to the issuer at par value on designated dates.

5. **Convertible bond** – giving the bondholder the right to exchange the bond for a specified number of shares.
   * Bondholder can take advantage of the future growth of the issuer’s company.
   * Issuer can raise capital at a lower cost.

6. **Exchangeable bond** – allows bondholder to exchange the issue for a specified number of common stocks of another corporation.
**US Government bonds**

*US Treasury bills* – issued in denomination of $10,000 or more with fixed terms to maturity of 13, 26 and 52 weeks; zero coupon and sold on discount basis, they are highly liquid and sold at auction.

*US Treasury notes* – have maturities of 1 to 10 years and are sold in denominations as small as $1,000; coupon payment (fixed throughout the life) paid every 6 months until maturity; sold at auction.

*US Treasury bonds* – with maturities of more than 10 years; they make coupon payments and are callable (redeem the bond for its face value).
Municipal bonds
Issued by agencies of state and local governments
General obligation bonds
Backed by a government body
Revenue bonds
Backed by revenue generated by a project

Corporate bonds
Issued by corporations for the purpose of raising capital for operations & new ventures; some bonds are traded on an exchange, but most are traded over-the-counter (OTC markets) in a network of bond dealers
Risks associated with investing in bonds

*Interest rate risk*

The price of a typical bond will change in the opposite direction from a change in interest rates: as interest rates rise, the price of a bond will fall.

* The sensitivity of a bond’s price to changes in interest rates depends on coupon, maturity, etc.

*Default risk* (credit risk)

Risk that the issuer of a bond may default.

Bonds with default risk trade in the market at a price lower than comparable US Treasury securities.

Default risk is gauged by quality ratings.
## Rating classifications

<table>
<thead>
<tr>
<th></th>
<th>Moody's</th>
<th>Standard &amp; Poor's</th>
</tr>
</thead>
<tbody>
<tr>
<td>High grade</td>
<td>Aaa</td>
<td>AAA</td>
</tr>
<tr>
<td></td>
<td>Aa</td>
<td>AA</td>
</tr>
<tr>
<td>Medium grade</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>Baa</td>
<td>BBB</td>
</tr>
<tr>
<td>Speculative grade</td>
<td>Ba</td>
<td>BB</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Default danger</td>
<td>Caa</td>
<td>CCC</td>
</tr>
<tr>
<td></td>
<td>Ca</td>
<td>CC</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
</tr>
</tbody>
</table>

* **investment grade** – either high or medium grade

* **junk bonds** – in or below speculative grade
Other risks of bonds

*Inflation risk* (purchasing–power risk)
- Variation in the value of cash flows from a bond due to inflation, as measured in terms of purchasing power.
- For all but floating–rate bonds, an investor is exposed to inflation risk.

*Exchange rate risk* – foreign currency denominated bonds.

*Liquidity* or *marketability risk* – measured by the size of bid-ask spreads.
Time value of money

Future value = present value x compounding factor

Present value = future value x discount factor

Compounding factor = \((1 + r)^n\), where
- \(r\) is the interest rate over one time period
- \(n\) is the number of interest earning periods.

Discount factor = \(1 / (1 + r)^n\).
Continuous compounding

Let $R$ be the interest rate per annum and $t$ be the total interest earning time period (in years).

Suppose there are $m$ compounding intervals per year, then

$$\text{compounding factor} = \left(1 + \frac{R}{m}\right)^m.$$ 

In the limit $m \to \infty$, compounding factor $= e^{Rt}$

For example, $R = 8\%$, $t = 2.5$ (years); the total interest earned for a principal $P_0$

$$P_0 \left(e^{0.08 \times 2.5} - 1\right) = P_0 \left(e^{0.2} - 1\right) \approx 0.2217P_0.$$
Suppose that a bond with face value $F$ makes $m$ coupon payments of $c/m$ each year and there are $n$ periods remaining. Let $P$ denote the current market price of the bond. The yield to maturity $\lambda$ is given by

$$P = \frac{F}{(1 + \frac{\lambda}{m})^n} + \sum_{k=1}^{n} \frac{c/m}{(1 + \frac{\lambda}{m})^k}$$

$$= \frac{F}{(1 + \frac{\lambda}{m})^n} + \frac{c}{\lambda} \left[ 1 - \frac{1}{(1 + \frac{\lambda}{m})^n} \right].$$

**Remark**
Here, we assume an exact number of coupon periods remaining. The price-yield formula requires adjustment for dates between coupon payment dates.
Yield to maturity (YTM) of a bond

A bond's yield is the *internal rate of return* of the bond at the current market price. Alternatively, it is the interest rate at which the present value of the stream of payments is exactly equal to the current price.

* Bonds of maturity of 30 years and the coupon rates
Price, yield, coupon and time to maturity

- The general interest rate environment exerts a force on every bond, urging its yield to confirm to that of other bonds.
- As yields move, bond prices move accordingly.
- The price change required to match a yield change varies with the structure of the bond (coupon rate and maturity).
- The price-yield curves are convex, and have negative slope (price and yield have an inverse relation).
- Price increases with coupon rate.
Two points on a price-yield curve that can be located.

i. When YTM = 0;
   price = 100 + coupon rate × years to maturity
   e.g. coupon rate = 3%, years to maturity = 30,
   then price = 190.

ii. When YTM = coupon rate;
    price = 100 = par value.

When the yield is exactly equal to the coupon rate, the bond is termed a *par bond*. 
Influence of maturity on price-yield curves

Bonds of 10% coupon rate but with different maturities

As maturity increases, the price-yield curve becomes steeper, indicating that longer maturity implies greater sensitivity of price to yield.
### Prices of 9% coupon bonds.

The prices of long-maturity bonds are more sensitive to yield changes.

<table>
<thead>
<tr>
<th>Time to maturity</th>
<th>5%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>103.85</td>
<td>100.94</td>
<td>100.00</td>
<td>99.07</td>
<td>94.61</td>
</tr>
<tr>
<td>5 years</td>
<td>117.50</td>
<td>104.06</td>
<td>100.00</td>
<td>96.14</td>
<td>79.41</td>
</tr>
<tr>
<td>10 years</td>
<td>131.18</td>
<td>106.80</td>
<td>100.00</td>
<td>93.77</td>
<td>69.42</td>
</tr>
<tr>
<td>20 years</td>
<td>150.21</td>
<td>109.90</td>
<td>100.00</td>
<td>91.42</td>
<td>62.22</td>
</tr>
<tr>
<td>30 years</td>
<td>161.82</td>
<td>111.31</td>
<td>100.00</td>
<td>90.54</td>
<td>60.52</td>
</tr>
</tbody>
</table>
Bondholders are subject to yield risk affecting the near-term value of the bond. An immediate loss is resulted if the bond is sold in near future.

On the other hand, the holder continues to receive the promised coupon payments and the face value at maturity (this is why bonds are called fixed income securities).
More properties on yield to maturity

There is not one yield offered on all bonds. The yield offered on a particular bond depends on 4 major factors:

1. A nominal return required to induce people to save (measured as the YTM on a US Treasury bond).

2. Compensation for default risk.

3. Adjustment for various options embedded in the bond such as the right to call.

4. Tax and liquidity features.
Summary of properties of bond pricing

1. When the annual coupon rate and YTM are identical, a bond will always sell at par.

2. Bond prices move inversely to changes in YTM.

$100

bond price

year to maturity

YTM = 4%

YTM = 6%

YTM = 8%

Price of 6% coupon bonds
3. Long term bonds are more price sensitive to a given change in the YTM than are shorter-term bonds.

4. While the price sensitivity of a bond increases with its maturity, this sensitivity increases at a decreasing rate.

5. Higher-coupon bonds are less price sensitive to a given YTM change than are lower-coupon bonds - “bird in the hand” argument. Indeed, a higher coupon bond has a shorter duration.