Anatomy of convertible bonds
Review of convertible formulas

stock price $30.00 per share
stock dividend $0.50 per share
convertible market price $1,000
coupon rate 7.00%
maturity 20 years
conversion price $36.37

Stock dividend yield = annual dividend rate / current stock price
= $0.50 / $30.00 = $1.67%
Conversion ratio
= number of shares for which one bond may be exchanged
= par / conversion price
= $1,000 / $36.37 = 27.50 shares

Conversion value
= equity value or stock value of the convertible
= stock price x conversion ratio
= $30.00 x 27.50 = $825.00
Conversion premium
= (convertible price – conversion value) / conversion value
= ($1,000 – $825.00) / $825.00 = 21.21%

Dollar premium
= convertible price – conversion value (expressed in points)
= ($1,000 – $825.00) / $1,000 x 100%
= 17.50 points
Conversion premium

In a **bullish** environment, the enthusiasm of the market **boosts** conversion premium levels.

- National Semiconductor Corporation (Sept 1995) – coupon rate 6.5 percent and conversion premium of 45 percent.
- 3Com Corporation (Nov., 1994) – coupon rate 10.25 percent and conversion premium of 70 percent. Bondholders are compensated with a **high** coupon rate while they wait for the stock price to rise.
Break even calculations

Break even (years)
= conversion premium / (convertible yield – stock yield)
= 21.21 / (7.00 – 1.67) = 3.98 (years)

This represents the number of years necessary for the stock investor to recover the conversion premium (extra cost of buying the convertible rather than the stock) from the convertible’s higher income relative to an instrument of an equivalent amount in the stock.

- After 3.98 years, the convertible has made up, in income alone, the amount of the conversion premium.
Break-even calculations (cont’d)

Dollar maintenance

\[
\text{Dollar maintenance} = \frac{\text{market price} - \text{conversion value}}{\text{coupon} - \left( \frac{\text{market price}}{\text{stock price}} \right) \text{ stock dividend}}
\]

It measures the time it takes for the convertible yield advantage to pay for its premium compared to an equivalent dollar amount purchased of the underlying stock.

- Some people may choose to use \textit{conversion ratio} instead of market price/stock price.
Weaknesses of break-even analysis

- It ignores the main advantage of convertible: protection on downside risk on the underlying equity.
- It ignores the margin of safety offered by the convertible with the payment of principal at maturity.
**Bond investment value**

- Present value of the interest and principal payments discounted at the straight (non-convertible) bond interest rate

\[
\text{bond interest value} = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{P}{(1+r)^n}
\]

where \( P = \text{par value}, \ r = \text{discount rate}, \ C = \text{coupon rate}, \ n = \text{number of periods to maturity}. \)

<table>
<thead>
<tr>
<th>Years</th>
<th>payment</th>
<th>present value factor</th>
<th>present value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 20</td>
<td>$80</td>
<td>8.514</td>
<td>$681.12</td>
</tr>
<tr>
<td>20</td>
<td>$1,000</td>
<td>0.149</td>
<td>$149.00</td>
</tr>
</tbody>
</table>

\[
\text{take } r = 10\%
\]

\[
\text{total present value} = $830.12
\]
Estimation of the discount rate

Use the yield-to-maturity of a similar non-convertible bond as a proxy.

- Ratings are not very responsive to changing financial fundamentals.
- The apparent deterioration of the creditworthiness of an issue will not be reflected in the convertible price because the common stock may be rising.
  - This is because the stock price reflects the increase in business, while the bond value reflects the increase in debt/equity ratio.
Duration

Duration is the weighted average of the times that the principal and interest payments are made.

$$\text{duration} = \frac{\sum_{t=1}^{n} tC_t / (1 + i)^t}{\sum_{t=1}^{n} C_t / (1 + i)^t}$$

where $t$ is the time of payment
$C_t$ is the coupon and/or principal payment
$i$ is the market yield.

Duration analysis provides a measure how bond values change with changing interest rates.
Duration analysis applied to convertibles

The approximate for the convertible bond’s interest rate sensitivity

$$D_{cv} = D_{adj} \left(1 - \frac{C}{I} \right)$$

where $C = \text{conversion value}$ and $I = \text{investment value}$.

• The equity component of the convertible bond may dampen the convertible’s interest rate sensitivity, depending on the bond’s equity participation. Hence, convertibles trading high above their investment value will be less sensitive to interest rates.
Duration and coupon

• For non-convertible bonds, the duration decreases as their coupon increases. This is because higher coupon bonds deliver more cash flows near the start of the bond’s life.

• With convertible feature, the higher coupon rate may lead to lower propensity to convert. The CB then has longer life, so this leads to higher duration.

These two effects are counteracting.
Correlation calculation

\[
\frac{dS}{S} = \mu_S dt + \sigma_S dZ_S \quad \text{and} \quad \frac{dr}{r} = \mu_r dt + \sigma_r dZ_r
\]

The expressions \(dZ_S\) and \(dZ_r\) are increments of a unit Brownian motion.

We are seeking the correlation of \(\sigma_S dZ_S\) and \(\sigma_r dZ_r\), which is the same as the correlation between \(d(\ln S)\) and \(d(\ln r)\).
Correlation calculation (cont’d)

Form the series \( x_i = \ln \frac{S_{i+1}}{S_i} \) and \( y_i = \ln \frac{r_{i+1}}{r_i} \). The sample mean and standard deviation of \( x_i \) (log of the return) are

\[
\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \text{and} \quad \sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu_x)^2}.
\]

The correlation coefficient between the two series

\[
\rho_{xy} = \sum_{i=1}^{n} \left( \frac{x_i - \mu_x}{\sigma_x} \frac{y_i - \mu_y}{\sigma_y} \right) / (n-1).
\]
Correlation with interest rates

Consider the impact of rise in interest rate

- The future share price is expected to be higher because of higher risk neutralized drift rate.
- Due to negative correlation between interest rate and share price (say, the S&P 500-stock index has a correlation of about minus 0.5), the share price drops first.

Negative correlations should lower CB value; positive correlations should make it worth more.

Typical CBs may have price differences in the range of 15-20% when correlations move from 1.0 to −1.0.
Pricing simple convertible bonds

Assumptions

• Non-callable and can be converted only at maturity.
• Markets are perfect and dynamically complete so that the standard contingent claims pricing approach can be used. No transaction costs and no bankruptcy costs.
• Capital structure consists of common shares and non-callable convertibles.
\( N = \) number of common shares
\( M = \) number of convertibles
\( f = \) face value per convertible
\( F = \) face value of the convertible issue = \( Mf \)
\( r = \) conversion ratio

Upon conversion, the convertible holders will possess \( \lambda \) fraction of equity, where

\[
\lambda = \frac{Mr}{N + Mr}.
\]

The parameter \( \lambda \) is called the \textit{dilution factor} of the convertible issue.
Let $V_T$ denote the value of the firm at maturity after the last coupon has been paid. The holder will convert if and only if

$$\lambda V_T > F$$

Value of convertible bond at maturity

$$V_T = \begin{cases} V_T & \text{if } V_T < F \\ F & \text{if } F < V_T < F / \lambda \\ \lambda V_T & \text{if } V_T > F / \lambda \end{cases}$$

$$= \min(F, V_T) + \lambda \max(V_T - F / \lambda, 0).$$
Decomposition into a straight bond plus $\lambda$ units of call with strike $F/\lambda$
Contingent claim approach

Wide spread use of the option pricing theory for pricing convertibles.
* Overall assessment of the price impact of different features in a convertible.

If investment bankers try to take advantage of the demand by overpricing the issue, the demand faded fast.
Risks associated with convertible bond investment

*Interest rate risk*
The price of a typical bond changes in the opposite direction from a change in interest rate.

*Call risk*
1. The cash flow pattern of a callable bond is not known with certainty.
2. Since the issuer will call the bond when the interest rates have dropped, the investor is exposed to reinvestment risk.
3. The capital appreciation potential of a bond will be reduced since the bond price is capped by the call price.

*Liquidity risk and default risk*