Fina556 – Structured Products and Exotic Options

Topic 4 — Mortgage backed securities and structured credit products

4.1 Mortgage backed securities

4.2 Synthetic collateralized debt obligations

4.3 Constant proportional debt obligations

4.1 Mortgage backed securities

- A *mortgage loan* is a loan secured by the collateral of some specific real estate property which obliges the borrower to make a predetermined series of payments.

- A *mortgage design* is a specification of the interest rate, term of the mortgage, and manner in which the borrowed funds are repaid.

- *Mortgage originator* (original lender) can either
  - hold the mortgage in their portfolio
  - sell the mortgage to an investor or
  - use the mortgage as collateral for the issuance of a security (mortgage backed security).

Contract rate (interest rate on mortgage loan)

The contract rate is greater than the yield on a Treasury security of comparable maturity. The spread reflects

- cost of servicing
- costs associated with default (not eliminated despite the collateral)
- poorer liquidity
- uncertainty concerning the timing of the cash flow
  - prepayment risk that leads to reinvestment of funds at a lower interest rate.

Mortgage instalments

- The borrower pays interest and repays principal in equal instalments over an agreed upon period of time (term of the mortgage). The frequency of payment is typically monthly.

- The servicing fee is a portion of the mortgage rate. The interest rate that the investor receives is called the *net coupon*.

Growing equity mortgages

- It is a fixed-rate mortgage whose monthly mortgage payments increase over time.

Amortization schedule for a level-payment fixed-rate mortgage

Mortgage loan:	$\$100,000$
Mortgage rate:	8.125% per annum
Monthly payment:	\$742.50
Term of loan:	30 years (360 months)

$$\text{monthly payment } P = \text{mortgage balance} \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

where i is the monthly interest rate.

Example

$n = 360$, mortgage balance $M = \$100,000, i = 0.08125/12$.

$$\text{Mortgage payment } M = \$742.50.$$

Proof of the mortgage formula

$$P[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1] = M(1+i)^n$$

extend one extra period:

$$P[(1+i)^n + (1+i)^{n-1} + \cdots + (1+i)] = M(1+i)^{n+1}.$$

Subtract the two terms:

$$P[(1+i)^n - 1] = Mi(1+i)^n$$

so that

$$P = \frac{Mi(1+i)^{n+1}}{(1+i)^n - 1}.$$

Month	Beginning Mortgage Balance	Monthly Payment	Monthly Interest	Principal Repayment
1	100,000.00	742.50	677.08	65.41
2	99,934.59	742.50	676.64	65.86
3	99,868.73	742.50	676.19	66.30
⋮	⋮	⋮	⋮	⋮
358	2,197.66	742.50	14.88	727.62
359	1,704.05	742.50	9.95	732.54
360	737.50	742.50	4.99	737.50

- Interest portion declines and repayment portion increases.

Adjustable rate mortgages

The mortgage rate is reset periodically in accordance with some chosen reference rate.

Other terms

- Rate caps limit the amount that the contract rate may increase or decrease at the reset date.
- A lifetime cap sets the maximum contract rate over the term of the mortgage loan.

Prepayment

Payments made in excess of the scheduled principal repayments. The amount and timing of the cash flows from the repayments are not known with certainty.

- Sale of a home
- Market rates fall below the contract rate
- Failure to meet the mortgage obligations

Factors affecting prepayment behaviors

1. Prevailing mortgage rate – the current level of mortgage rates relative to the borrower's contract rate.

   - The spread should be wide enough to cover the refinancing costs.

2. Path history of rate spread is important

   – depends on whether there have been prior opportunities to refinance since the underlying mortgages were originated.

3. Presence of prepayment penalty.
4. Macroeconomic factors e.g. growing economy results in a rise in personal income and opportunities for worker migration.
5. Seasonal factor: Home buying increases in Spring and reaches a peak in the late summer. Since there are delays in processing the contracts, the peak may not be observed until early fall.

Interest rate path dependence

Prepayment burnout – Prepayments are path dependent since this month's prepayment rate depends on whether there have been prior opportunities to refinance since the underlying mortgages were originated.

Example – path of interest rates in the past 3 years

First path: $11\% \to 8\% \to 13\% \to 8\%$

Second path: $11\% \to 12\% \to 13\% \to 8\%$

More refinancing occurs now when the interest rates follow the second path.

Prepayment models

Describes the expected prepayments on the underlying pool of mortgages at the current time in terms of the prevailing yield curve and other relevant variables.

– predicted from an analysis of the historical data.

Example

Weekly report "Spread Talk" published by the Prudential Securities

- provides the 6-month, 1-year and long-term prepayment projections assuming different amounts of shift in the interest rates.

Mortgage-backed securities (MBS) are securities backed by a pool of mortgage loans.

1. Mortgage passthrough securities;

2. Collateralized mortgage obligations;

3. Stripped mortgage-backed securities.

The last two types are called *the derivative mortgage-backed* securities since they are created from the first type.

MBS versus fixed income investments

- Virtually no default risk since the mortgages in a pool are guaranteed by a government related agency, such as GNMA (Government National Mortgage Association) or FNMA (Federal National Mortgage Association).

- Originating a mortgage loan is like writing a coupon bearing bond, except that the par is repaid in amortized amount periodically.

- Prepayment risk – like the call right of a bond issuer

  Prepayment privileges given to the householder to put the mortgage back to the lender at its face value.

Default and mortgage guarantees

Default probability is closely related to

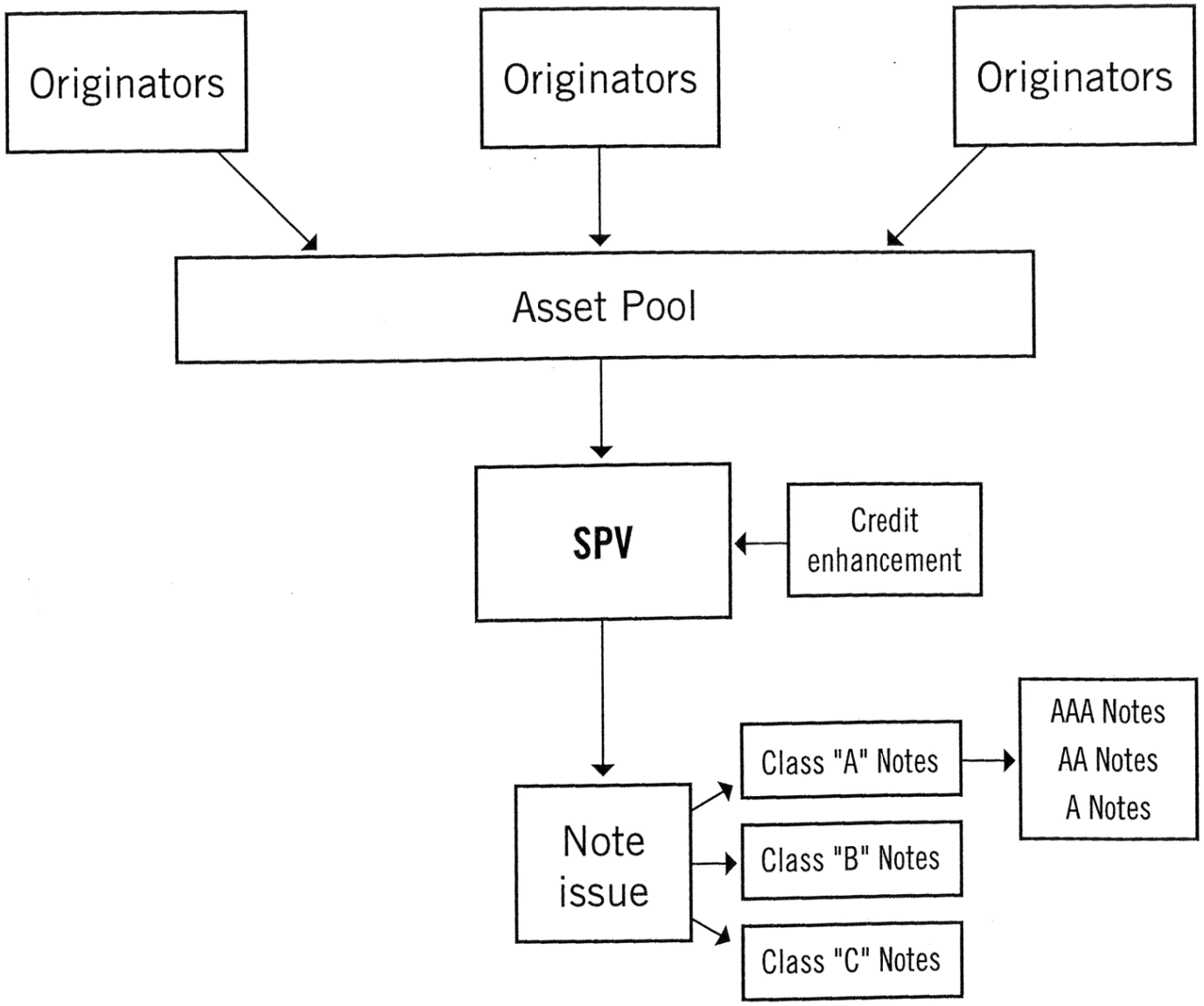
(i) Loan to property value ratio

(ii) ratio of mortgage payment to income.

Guarantees from either the private mortgage insurance companies or government agencies.

- Risk of a major downturn in the economy cannot be diversified away by private insurance.

Securitization Process



Mortgage passthrough securities

- A mortgage passthrough security is a security created when one or more holders of mortgages form a pool of mortgages and sell shares or participation certificates in the pool.

- The cash flows consist of the monthly mortgage payments representing interest, scheduled repayment of principal, and any prepayments.

- Payments are made to security holders each month. The monthly cash flows for a passthrough are less than the monthly cash flows of the underlying mortgages by an amount equal to servicing and other fees.

- Not all of the mortgages included in the pool that are securitized have the same mortgage rate and the same maturity. A weighted average coupon rate and a weighted average maturity are determined.

Senior/Subordianted structures

- The subordinated class is the first-loss piece absorbing all losses on the underlying collateral, thus protecting the senior class.

- The senior class is giving up yield to the subordinated class holders.

Example

$100 million deal divided into

$92,25 million senior class

$7.75 million subordinated class

Suppose there is $10 million of losses, the subordinated class experiences $7.75 million of losses (100% loss) and the senior class experiences a loss of $2.25 million (2.4% = $2.25 / $92.25 loss).

Contraction risk – decline in mortgage rates

Suppose an investor buys a 10% coupon Ginnie Mae at a time when mortgage rates are 10%. What would be the impact on prepayments if mortgage rates decline to 6%.

- The price of an ordinary option free bond will rise as the rate declines, but in the case of passthrough security the rise in price is less because there is a higher prepayment. The upside price potential is truncated due to prepayments. The cash flows from the prepayments can only be reinvested at a lower rate.

Expansion risk – rise in mortgage rates

What would happen if the mortgage rates rise to 15%?

- The price of the passthrough, like the price of any bond, will decline.
- It declines more because the higher rates will tend to slow down the rate of prepayment, in effect increasing the amount invested at the coupon rate, which is lower than the market rate.

Collateralized mortgage obligations

A collateralized mortgage obligation is a debt instrument collateralized by the mortgage passthrough certificates. The cash flows (interest and principal) are directed to different bond classes, called tranches so as to mitigate different forms of prepayment risk.

- The creation of a CMO *cannot eliminate the prepayment risk*. It serves only to redistribute prepayment risk among different classes of bond holders.

- CMO class has a different coupon rate from that for the underlying collateral, resulting in instruments that have varying risk-return characteristics that fit the needs of various fixed-income investors.

- Assume that investors have different preferred maturities and so they should be willing to pay different prices for securities of different expected maturities.

Five-tranche sequential-pay structure with floater, inverse floater, and accrual bond classes*

Tranche	Par Amount	Coupon Rate (%)
A	$\$194,500,000.00$	7.50
B	$\$36,000,000.00$	7.50
FL	$\$72,375,000.00$	1-month LIBOR $+0.5$
IFL	$\$24,125,000.00$	$28.50 - 3\times$(1-month LIBOR)
Z(accrual)	$\$73,000,000.00$	7.50
	$\$400,000,000.00$	

Note that the weighted average of the coupon rates of the FL and IFL tranches is

$$\begin{aligned} & \frac{1}{4}(28.50 - 3 \times \text{1-month LIBOR}) \\ + \ & \frac{3}{4}(\text{1-month LIBOR} + 0.5) = 7.5\%. \end{aligned}$$

* The interest for the *accrual tranche* would accrue and be added to the principal balance (like a zero-coupon bond). The interest that would have been paid to the accrual bond class is used to speed up paydown of the principal balance of the earlier bond classes.

- The Z bonds are more risky to interest rate fluctuations and default risk since

  (i) they have long effective maturities;
  (ii) they suffer more from the uncertainty about the timing of prepayment;
  (iii) the losses in principal payments are absorbed since it is the tranche which receives the principal payments last.

Payment rules

1. *For disbursement of principal payments:*

   - Disburse the principal payments to tranche A until it is paid off completely.
   - After tranche A is paid off completely, disburse the principal payments to tranche B until it is paid off completely.
   - After tranche B is paid off completely, disburse the principal payments to tranches FL and IFL until they are paid off completely.
   - The principal payments between tranches FL and IFL should be made in the following way: 75% to tranche FL and 25% to tranche IFL.
   - After tranches FL and IFL are paid off completely, disburse the principal payments to tranche Z until the original principal balance plus accrued interest is paid off completely.

2. *For payment of periodic coupon interest:*

   - Disburse the periodic coupon interests to tranches A, B, FL, and IFL on the basis of the amount of principal outstanding at the beginning of the period.
   - For tranche Z, accrue the interest based on the principal plus accrued interest in the preceding period.
   - The interest for tranche Z is to be paid to the earlier tranches as a principal paydown.
   - There is a cap on FL and a floor on IFL. The maximum coupon rate for FL is 10% ; the minimum coupon rate for IFL is 0%. The factor 3 in IFL is called the coupon leverage.

Five tranche sequential pay with an accrual tranche and an interest only tranche

The excess interest between the coupon rate on the tranches and the coupon interest on the collateral is paid to the interest-only tranche.

Tranche	Par Amount	Coupon Rate (%)
A	$\$194,500,000.00$	6.00
B	$\$36,000,000.00$	6.50
C	$\$96,500,000.00$	7.00
Z	$\$73,000,000.00$	7.25
IO	$\$52,566,667$ (notional)	7.50
	$\$400,000,000.00$	

For the IO class, there is no par amount. The amount shown is the amount on which the interest payments will be determined. This is called the notional amount.

$$\text{Notional amount for 7.5\% IO} = \frac{\text{tranches par value} \times \text{excess interest}}{0.075}.$$

- The IO class is almost like an annuity with a lower counterparty no loss upon default of principals since the interest payments are generated by excess interest between the tranches' coupon rate and the interests from the collateral.

Tranche	Par Amount	Excess Interest (%)	Notional Amount for a 7.5% Coupon Rate IO
A	$\$194,500,000.00$	1.50	$\$38,900,000$
B	$\$36,000,000.00$	1.00	$\$4,800,000$
C	$\$96,500,000.00$	0.50	$\$6,433,333$
Z	$\$73,000,000.00$	0.25	$\$2,433,333$
		Notional amount for 7.5% IO	$\$52,566,666$

- An an illustration, in Tranche A, we observe

$$\$194,500,000 \times 1.5\% = \$38,900,000 \times 7.5\%.$$

Payment rules

1. *For payment of periodic coupon interest:*

   - Disburse periodic coupon interest to tranches A, B, and C on the basis of the amount of principal outstanding at the beginning of the period.

   - For tranche Z, accrue the interest based on the principal plus accrued interest in the preceding period. The interest for tranche Z is to be paid to the earlier tranches as a principal paydown.

   - Disburse periodic interest to the IO tranche based on the notional amount at the beginning of the period.

2. *For disbursement of principal payments:*

   - Disburse the principal payments to tranche A until it is paid off completely.
   - After tranche A is paid off completely, disburse the principal payments to tranche B until it is paid off completely.
   - After tranche B is paid off completely, disburse the principal payments to tranche C until it is paid off completely.
   - After tranche C is paid off completely, disburse the principal payments to tranche Z until the original principal balance plus accrued interest is paid off completely.

Why CMO are popular?

1. The CMO converts a long-term monthly payment instrument into a series of semi-annual payments, which are bond-like securities with short, intermediate and long maturities.

2. The multiple-maturity structure reduces the degree of uncertainty of cash flows for any particular maturity class, and provides the longer maturity classes with a higher degree of prepayment protection. This is because the shorter tranches absorb the initial burden of excess principal repayments.

3. Investors are attracted by the broader range of investment maturities made possible by the CMO structure. For example, insurance companies purchase heavily in the 4-6 year life tranche. Pension funds have been active in the longer tranche sector.

4. Credit quality

   The high quality of the collateral (GNMA etc.) along with the protective structure of the trust, enables these securities to generally carry the highest investment grade credit rating. CMOs are also free from events that cause price fluctuations in the corporate world.

5. Yield

   Offer the investors an attractive yield premiums over the Treasury and even some corporate bonds.

6. Liquidity

   The market is large and thus very liquid: similar to Treasury bonds.

Stripped mortgage backed securities

They are created by altering the distribution of principal and interest from the collateral. For example, all the interest is allocated to the IO class (interest only) and all the principal to the PO class (principal only).

- PO securities are purchased at a substantial discount from par value. The faster the prepayments, the higher the yield the investor will realize.

- IO investors want prepayments to be slow. This is because when prepayments are made, the outstanding principal declines, and less dollar interest is received.

Impact of mortgage rates

Falling interest rate – faster prepayment

- Principal payments are received sooner and discounted at a lower interest rate, the value of POs rises
- Earlier principal payments reduce interest payments to IOs. These interest payments are discounted at a lower interest rate, partially offsetting the decline in payments.

Rising interest rate – slower prepayment

- For POs, principal payments are deferred and discounted at higher interest rates, the value of POs drops.
- For IOs, interest received is higher than anticipated, though discounted at a higher interest rate. IOs can increase in value unless interest rates increase sharply.

Valuing MBS using Monte Carlo simulation

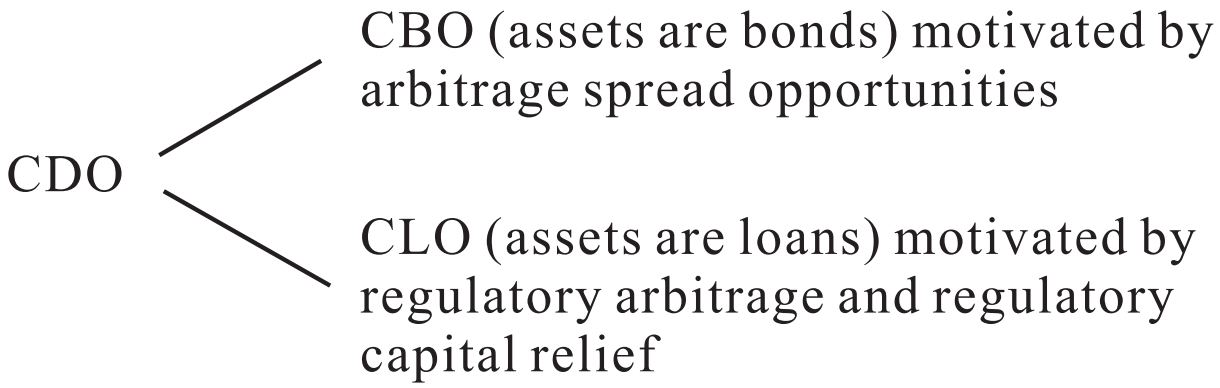
- Generate the random interest rate paths by taking as input today's term structure of interest rates and a volatility assumption.

- Prepayments are projected by feeding the refinancing rate and loan characteristics into a prepayment model. Given the projected prepayments, the cash flow along an interest rate path can be determined.

The simulation works by generating many scenarios of future interest rate paths. An estimate of the value of the MBS is the average of the sample values over many simulation trials.

4.2 Synthetic collateralized Debt Obligations

A *collateralized debt obligation* (CDO) is a security backed by a pool of assets security (e.g. corporate bonds, bank loans).



A CDO cash flow structure allocates interest income and principal repayments from a collateral pool of different debt instruments to a prioritized collection of CDO securities, called *tranches*.

A CDO is a way of creating securities with widely different risk characteristics from a portfolio of debt instruments. CDOs represent a major asset class of asset based securities.

Regulatory capital

Regulatory wedge – what market requires (economic capital) and what regulators require (regulatory capital)?

Loans are 100% risk weight items and capital charges of 8% are levied on them.

- Forcing banks to allocate the same quantity of capital to support a loan to an AA-rated company as to a B-rated company. This would bias the investment decision in favor of the B-rated loans.
- This results in "pricing distortions" since under such rule the capital costs of a loan are independent of the credit quality of the borrower.

Economics of CDOs

CDOs address some important market imperfections.

- Regulatory arbitrage and capital relief
- Illiquidity leading to a reduction in their market values

The uncertainty regarding interest and principal payments to the CDO tranches is determined mainly by the number and timing of defaults of the collateral securities.

Arbitrage spread opportunities

- The assets in the collateral pool are priced on a single asset basis.
- The tranching of notes really is a tranching of the loss distribution of the collateral pool. Since diversification decreases the risk of a portfolio, so that the price of the portfolio risk would be lower than the price obtained by just summing up exposure-weighted single risk.
- The spreads paid to notes investors are lower than the spreads earned on the bonds in the collateral pool.

In summary, the total spread collected on single credit risky instruments at the asset side exceeds the total "diversified" spread to be paid to investors on the tranched liability side.

Tranches

- Investor's proceeds (principal and interest) are paid from cash flows generated by the performance risk of the collateral pool.
- Different investors have different risk appetite, the notes issued by an SPV are tranched into different risk classes.

First loss piece (equity piece) — most subordinated tranche, receiving interest and principal payments only if all other note investors recovered their contractually promised payments.

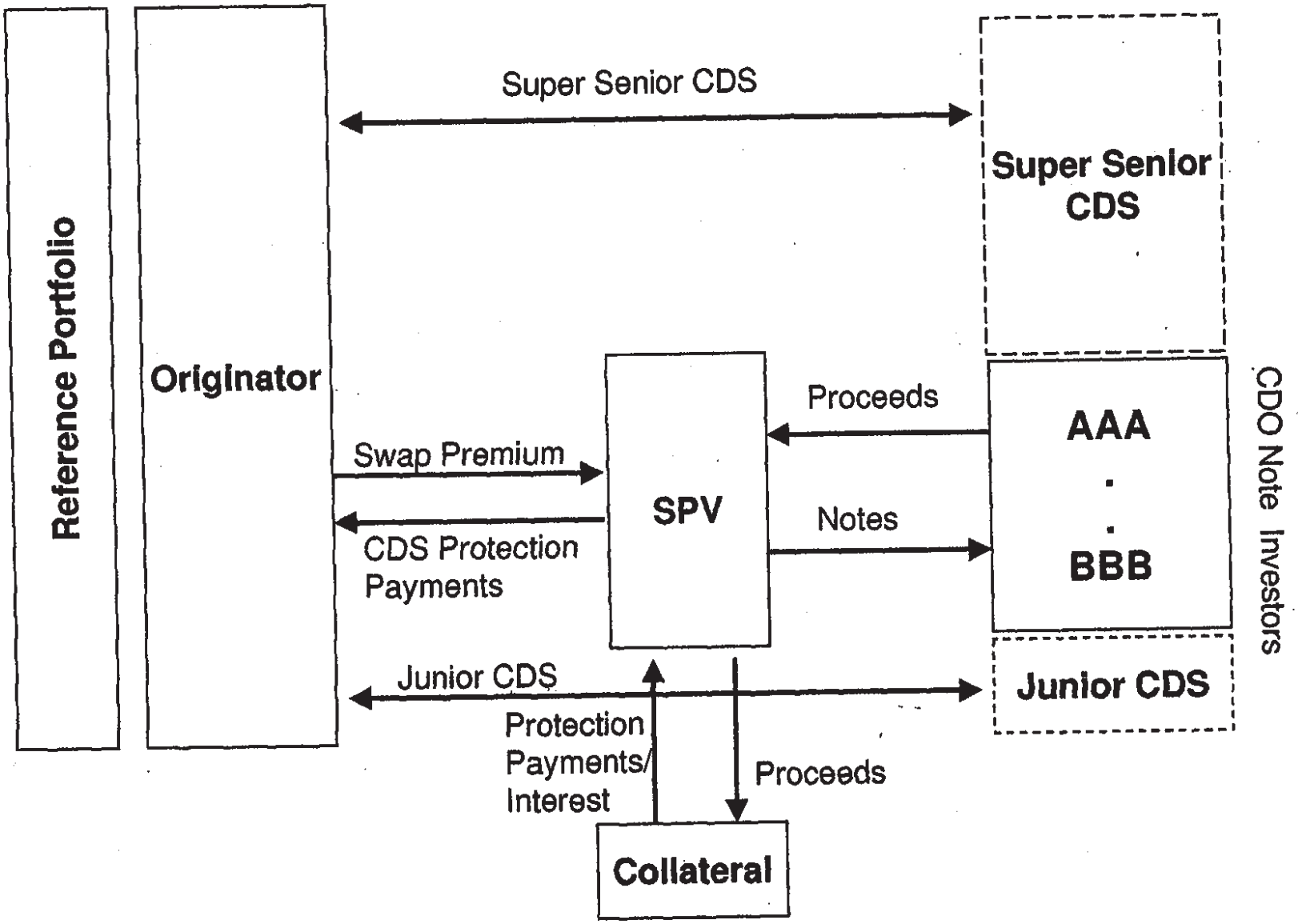
Junior, mezzanine and senior tranches

Receiving interest and principal proceeds in the order of their seniority: senior note holders receive their payments first, more junior note investors receive payments only if more prioritized payments have been made.

Additional parties involved in a CDO transaction, including

- rating agencies, which assign ratings to the issued notes,
- a trustee, which takes care that the legal documentation is honored and receives monthly trustee reports regarding the current performance of the structure
- some swap counterparties in case interest or currency risk has to be hedged, and
- lawyers, structuring experts, and underwriters at the beginning of the transaction.

Example of a synthetic CDO transaction.



Key features of Synthetic CDO's

- CDO entity does not actually own the pool of assets.
- Credit default swaps allow the transfer of economic risk (synthetic exposure) but not the legal ownership of the underlying securities.

Credit risk is distilled from a reference portfolio of loans, then channeled to the credit markets.

- Create a special purpose vehicle (bankruptcy-remote from the originating bank) that issues the credit-linked notes.
- Credit-linked notes will be collateralized by AAA-rated securities, that is, they are the obligations of a fully collateralized SPV.

Credit linked notes (CLNs) in a synthetic CDO

- The interest from the investment grade security and the periodic swap payments received from the default swap buyer are passed on to the CLN investors in the form of a yield on the notes.

- The CLN issuer is protected from default risk of the originator.

- Higher return for investors without directly getting into the credit derivatives market – same as buying a riskless floating rate note and selling a credit protection through a CDS.

- Conventional stream of cash flows – periodic fix/float coupons and principal at redemption, if no credit events occurs.

- The cash flows are altered upon the occurrence of a credit event experienced by a reference credit.

Moral hazard – asymmetric information

- Cherry picking – sorting assets into the portfolio pool based on the issuer's private information.
- In virtually every synthetic CDO and CLN, the 'buyer' of protection determines whether a credit event has occurred in the reference portfolio. Also the 'buyer' calculates the severity of its losses following a credit event, and how much the SPV will be required to pay under the swap.

Moody's advice – Good faith of the originator

No matter how carefully the transaction is structured, an aggressive protection buyer can interpret credit events more broadly than the seller intended, or obtain pricing for defaulted obligations that is unrealistic or not meaningful.

Olan Enterprises

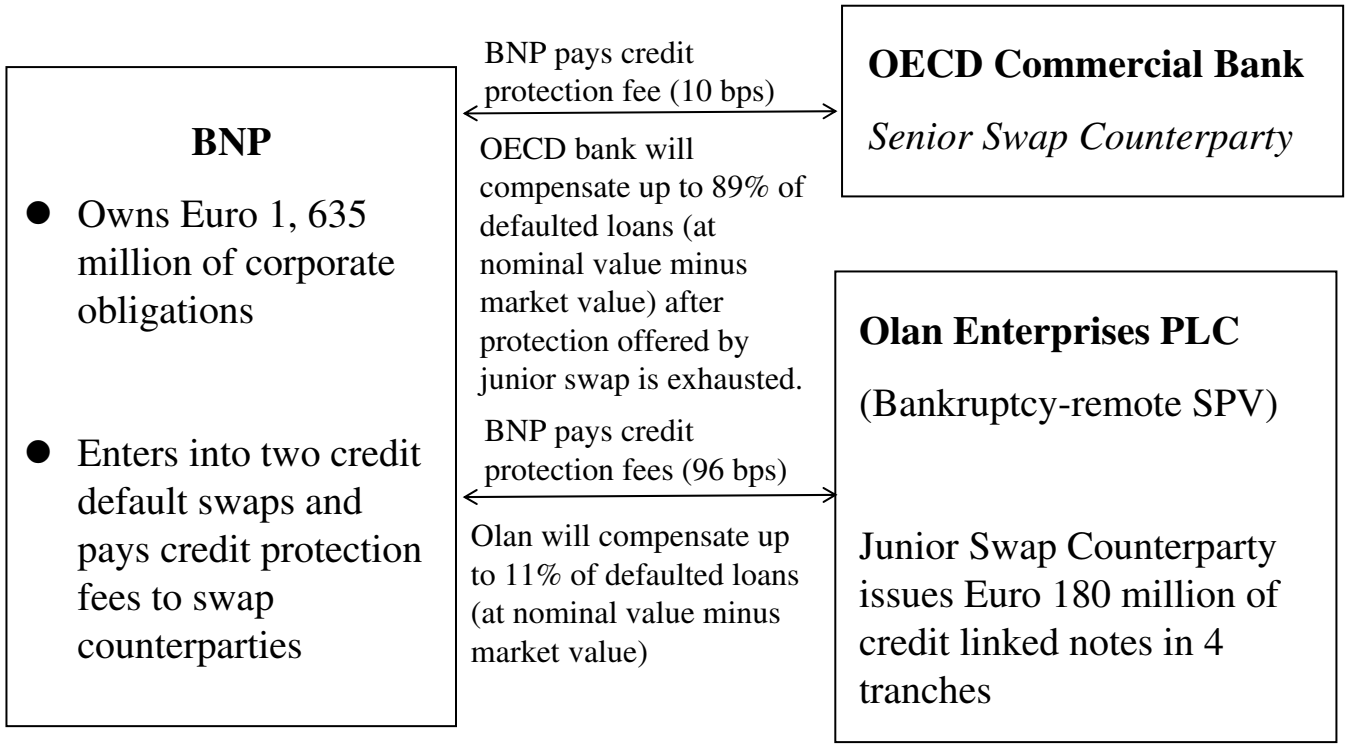
1. Olan financed its commitment under the junior credit default swap by issuing Euro 180 million of credit linked notes in 4 classes (11% of the reference portfolio). This is a partially funded synthetic CDO. Here, 11% of the capital is under the funded tranches while the remaining 89% is under the unfunded senior credit default swap. The goal of partial funding is to deliver favorable capital requirement without the funding cost disadvantage problem (comparing the funding costs of BNP and the tranche investors).

2. Olan used the proceeds from the notes to purchase 5-year French Treasury bonds (OATs) as collateral. Should a credit event occur, Olan must sell OATs to pay BNP's loss.

3. Olan receives the premium from the junior credit default swap. The fee, plus the coupon of the AAA-collateral, funds Olan's interest obligations on the credit linked notes.

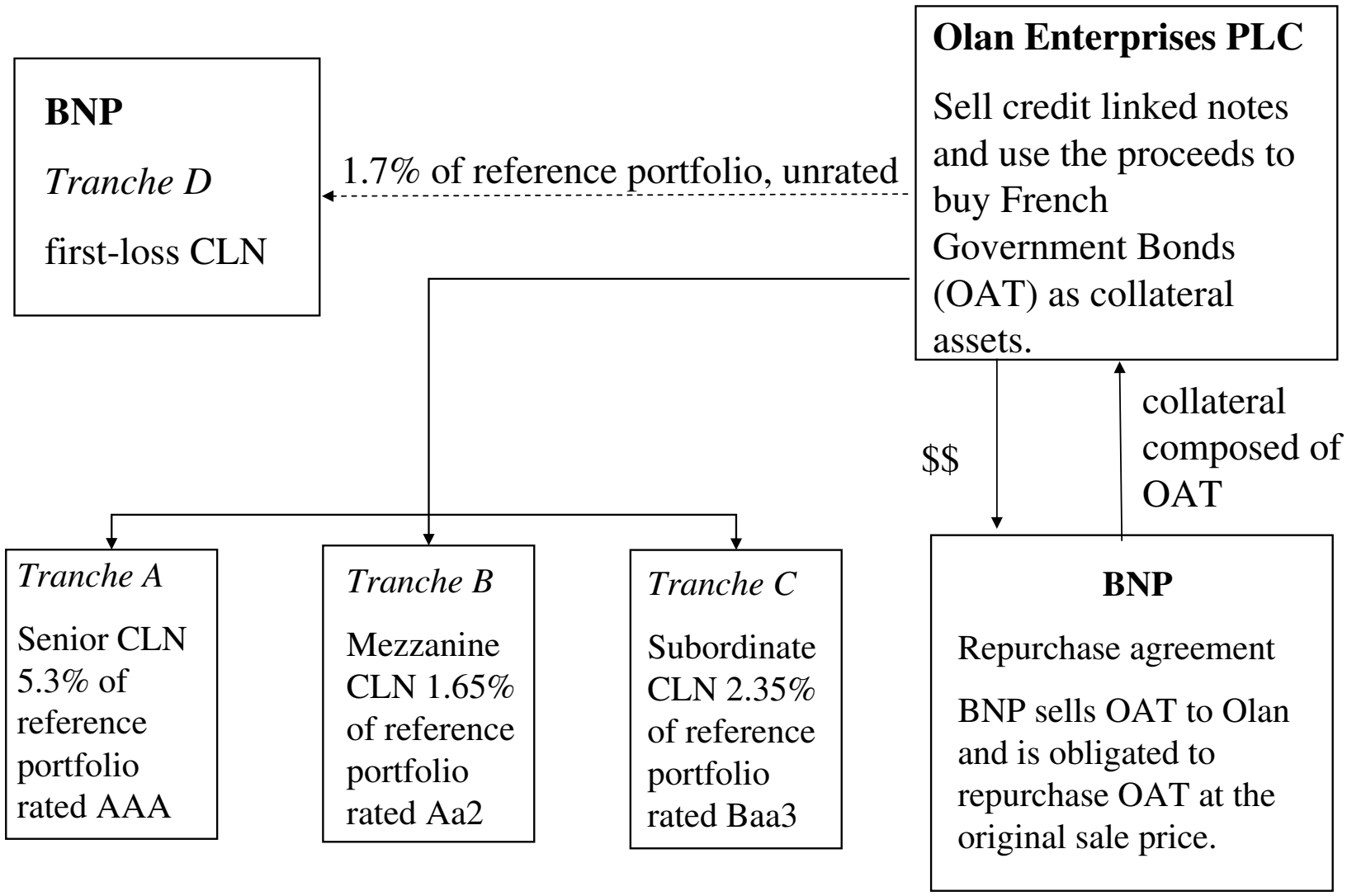
Olan 1 Transaction structure

Launched by Banque Nationale de Paris (BNP) in 1999.



Total premium (as percentage of reference portfolio) $= 0.96\% \times 11\% + 0.10\% \times 89\% = 0.1946\%$. Note that 180 million = 1,635 million $\times 11\%$.

Olan 1 Transaction structure



Credit linked notes – public issues

	Class A	Class B	Class C	Class D
Rating	AA	A	BBB	unrated
Amount (Euro)	86.66m	26.97m	38.42m	27.96m
bp over 3-month Euro-bor	30	40	150	NA
% of corporate credit exposure	5.3%	1.65%	2.35%	1.7%

Class D will absorb the first loss experienced by the reference portfolio. This first-loss CLN was retained by BNP.

Funded tranches

The tranche investor pays the notional amount of the tranche at the beginning of the deal and any defaults cause a writedown of the principal.

Transfer of credit risks

The 4 credit-linked notes have different exposures to credit risk.

- Class D funds the first level of losses (retained by BNP)

The credit risk beyond that funded by the SPV is shifted to an Organization for Economic Cooperation & Development (OECD) bank via a *senior* default swap.

The embedded risks in the reference portfolio of loans are shifted without having to sell the underlying loans – *synthetic CLO*.

OATs as collateral

OATs are used as collateral, first for the credit protection of BNP since the losses in defaulted loans are repaid by the collateralized OATs, then for the repayment of classes A, B, C & D. The AAA-rated OATs serve to ensure that defaulted loans up to 11% covered by the Junior CDS can be compensated fully by the sale of the OATs.

Repurchase agreement (mitigate the market risk associated with liquidation) BNP is committed to repurchase the OATs sold to Olan at the original price paid by Olan.

Costs

The originating bank has to pay coupons to note investors, credit swap premiums, unfront costs (rating agencies, lawyers, structuring and underwriting costs), ongoing administration costs, etc.

Regulatory capital for synthetic CLOs

$D =$ sponsoring bank's first loss
(class D retained by sponsoring bank)

$20\% =$ risk-weight assigned to the notional amount of
the senior credit swap

Senior $=$ notional amount of senior credit swap

$K_{fed} =$ capital requirement for the sponsoring US bank

$= \max(D, 0.08 \times (D + 0.2 \times \text{ senior } + 0 \times \text{ junior}) = 1.7\%$

$K_{CB} =$ capital requirement for the sponsoring bank under Commission
Bancaire (French banking regulator) regulations

$= D + 8\% \times 20\% \times \text{ senior} = 3.124\%$

The US K_{fed} is too lenient as compared to the French K_{CB}.

Impact of default correlation on tranche values

- Though the equity and mezzanine tranches contain a small fraction of the notional amount of the CDO's reference portfolio, they bear a majority of the credit risk.

- Warning – The leverage that the mezzanine tranches possesses implies their risk can be many times that of an investment-grade corporate bonds. The mezzanine tranche, in a second-loss position, suffers no losses in a boom and minimal loss in a trend growth scenario, but suffers most of the portfolio's expected loss in a recession. Therefore, mezzanine tranches are leveraged bets on the business cycle risk.

- Higher correlation makes the extreme case of very few defaults more likely, so the value of the equity tranche rises as correlation rises.

Stylized CDOs

Consider a portfolio with m firms and define

$$\begin{aligned} Y_{t,i} &= \text{default indicator process of firm } i \\ &= \begin{cases} 1 & \text{if firm } i \text{ has defaulted up to time } t \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

The cumulative loss

$$L_t = \sum_{i=1}^{m} \delta_i e_i Y_{t,i}$$

where e_i is the exposure of firm i and δ_i is the corresponding loss fraction.

- We consider a CDO with k tranches, indexed by $k \in \{1, \cdots, K\}$, and characterized by *attachment points* $0 = K_0 < K_1 < \cdots < K_k \leq \sum_{i=1}^{m} e_i$.

- Initially, the notional is equal to $K_k - K_{k-1}$; it is reduced whenever there is a default event such that the cumulative loss ℓ falls in the layer $[K_{k-1}, K_k]$.

- The notional of tranche k at time t, $N_k(t)$, is given by

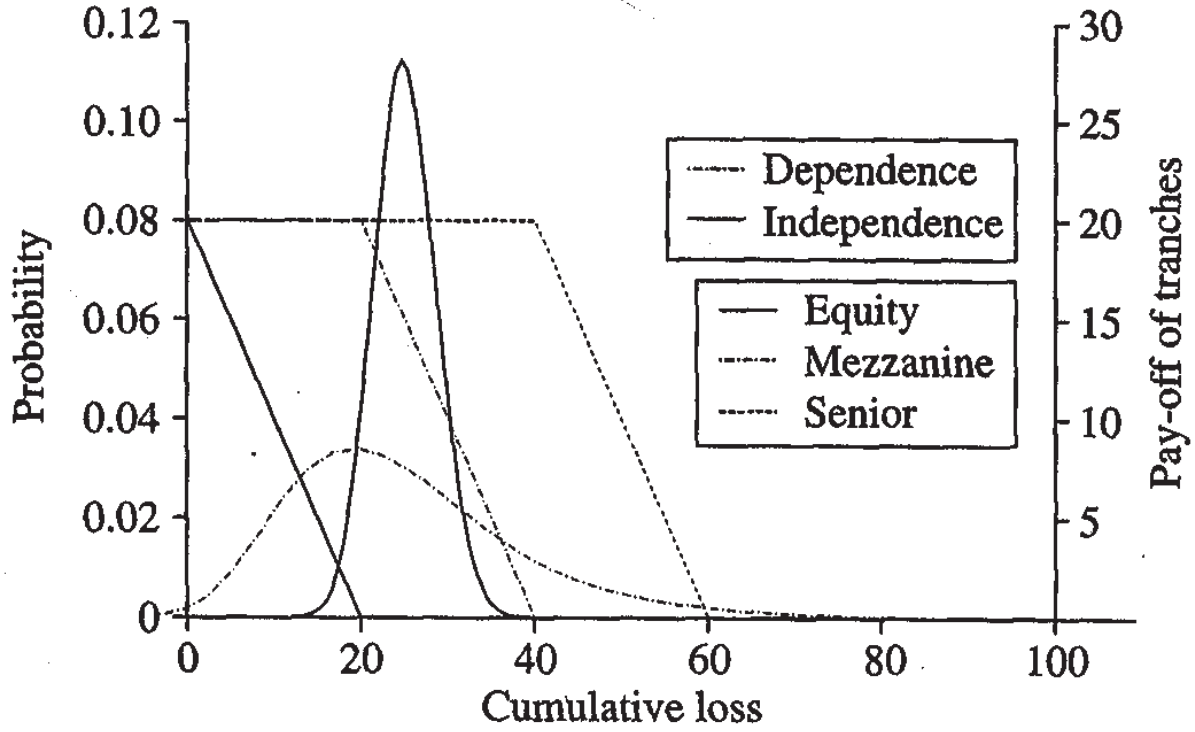
$$N_k(t) = f_k(L_t) \quad \text{with } f_k(\ell) = \begin{cases} K_k - K_{k-1}, & \text{for } \ell < K_{k-1}, \\ K_k - \ell & \text{for } \ell \in [K_{k-1}, K_k], \\ 0, & \text{for } \ell > K_k. \end{cases}$$

- Note that f_k can be written more succinctly as

$$f_k(\ell) = (K_k - \ell)^+ - (K_{k-1} - \ell)^+,$$

  i.e. the notional is equal to the sum of a long position in a put option on L_t with strike price K_k and a short position in a put with strike price K_{k-1}. Such positions are sometimes called a *put spread*.

- The probability distribution of the cumulative loss ℓ is highly dependent on the default correlation among the obligors. This is why the synthetic CDO tranches are called the credit correlation products.



Pay-off of a stylized CDO contract and distribution of the one-year loss L_1 for a default probability of 0.5% and different default correlations.

$m = 1,000$ firms, each with exposure of one unit.

$K_1 = 20, K_2 = 40, K_3 = 60$, corresponding to $2\%, 4\%$ and 6% of the overall exposure.

Independent defaults

- The one-year loss L_1 is typically close to its mean. Hence it is quite unlikely that a tranche k with lower attachment point K_{k-1} substantially larger than $E(L_1)$ (the senior tranche) suffers a loss, so the value of such a tranche is quite high.

- Since the attachment point K_1 on the equity tranche is typically lower than $E(L_1)$, it is quite unlikely that L_1 is substantially smaller than K_1, and the value of the equity tranche is low.

- The junior tranche $[2\% - 4\%]$ is expected to suffer some loss as $E(L_1)$ falls within the tranche. The loss may be lessened if the loss distribution is more flattened.

High default correlation

- If defaults are (strongly) dependent, diversification effects in the portfolio are less pronounced.
- Realizations with L_1 bigger than the lower attachment point K_2 of the senior tranche are more likely, as are realizations with L_1 smaller than the upper attachment point K_1 of the equity tranche.
- This reduces the value of tranches with high seniority and increases the value of the equity tranche compared with the case with (almost) independent defaults.

The synthetic CDO tranche spread depends on a number of factors.

- Attachment point: This is the amount of subordination below the tranche. The higher the attachment point, the more defaults are required to cause tranche principal losses and the lower the tranche spread.
- Tranche width: The wider the tranche for a fixed attachment point, the more losses to which the tranche is exposed.
- Portfolio credit quality: The lower the quality of the asset portfolio, measured by spread or rating, the greater the risk of all tranches due to the higher default probability and the higher the spread.
- Default correlation: If default correlation is high, assets tend to default together and this makes senior tranches more risky. Assets also tend to survive together making the equity safer.

Portfolio loss distribution

- No matter what approach we use to generate it, the loss distribution of the reference portfolio is crucial for understanding the risk and value of correlation products.

- The portfolio loss is clearly not symmetrically distributed. It is therefore informative to look at the entire loss distribution, rather than summarising it in terms of expected value and standard deviation.

- As correlation increases, the peak of the distribution falls and the high quantiles increase. The probability of larger losses increases and, at the same time, the probability of smaller losses also increases, thereby preserving the expected loss which is correlation independent.

- At maximum default correlation all the probability mass is located at the two ends of the distribution. The portfolio either all survives or its all defaults. It resembles the loss distribution of a single asset.

Tranches loss distribution

How does the shape of the portfolio loss distribution affect the pricing of tranches? Plot the loss distributions for a CDO with a 5% equity, 10% mezzanine and 85% senior tranches for correlation values of 20% and 50%.

Equity tranche

- At 20% correlation, we see that most of the portfolio loss distribution is inside the equity tranche, with about 14% beyond, representing the probability at 100% loss.
- As correlation goes to 50% the probability of small losses increases while the probability of 100% losses increases only marginally. Clearly equity investors benefit from increasing correlation.
- Equity tranche investors are long correlation. When correlations go up, equity tranches go up in value.

Mezzanine tranche

In most cases mezzanine investors benefit from falling correlation – they are short correlation. However, the correlation directionality of a mezzanine tranches depends upon the reference portfolio and the tranche. In certain cases a mezzanine tranche with a very low attachment point may be a long correlation position.

Senior tranche

Senior investors also see the risk of their tranche increase with correlation as more joint defaults push out the loss tail. Senior investors are short correlation. If correlation increases, senior tranches fall in value.

4.3 Constant proportional debt obligations (CPDOs)

- CPDOs emerged in August 2006 (originator is ABN AMRO).
- It was described as a "holy grail of structured credit" – they were the only triple-A rated instrument offering an impressible 200bp above LIBOR.

Overview of structure

- A CPDO is a fixed income instrument that aims to pay the stated coupons by taking leveraged exposure to a notional portfolio of credit indices. It comprises of exposure to a Credit Index Portfolio and a cash deposit.
- The Credit Index Portfolio aims to generate sufficient returns to enable the coupon payments to be made. Use of leverage to enhance returns. Leverage is the ratio of the risky exposure to the amount of the rated liabilities (coupons and principal).

– High returns are achieved via rule-based leveraged credit strategies. Considered as an investment strategy rather than as an asset class.

– Price is not directly impacted by movements in correlation (not a correlation product) since the spread of the cumulative loss distribution is immaterial.

Basic elements

- The structure is fully funded. The investor purchases a note (say US$100 m) with a final legal maturity of 10 years. The Note pays a coupon of around LIBOR plus 200 bps pa.

- The note is rated AAA by the rating agency. However, the principal and interest of the note are not specifically guaranteed.

- Initially, the *assets* of a CPDO are only the proceeds received from the investors.

- On the *liability* side, the CPDO has to pay a fixed coupon during the life of the transaction and the principal back at maturity.

- Initially, there is a shortfall between the net value of the assets (NAV) and the net present value (PV) of the liabilities.

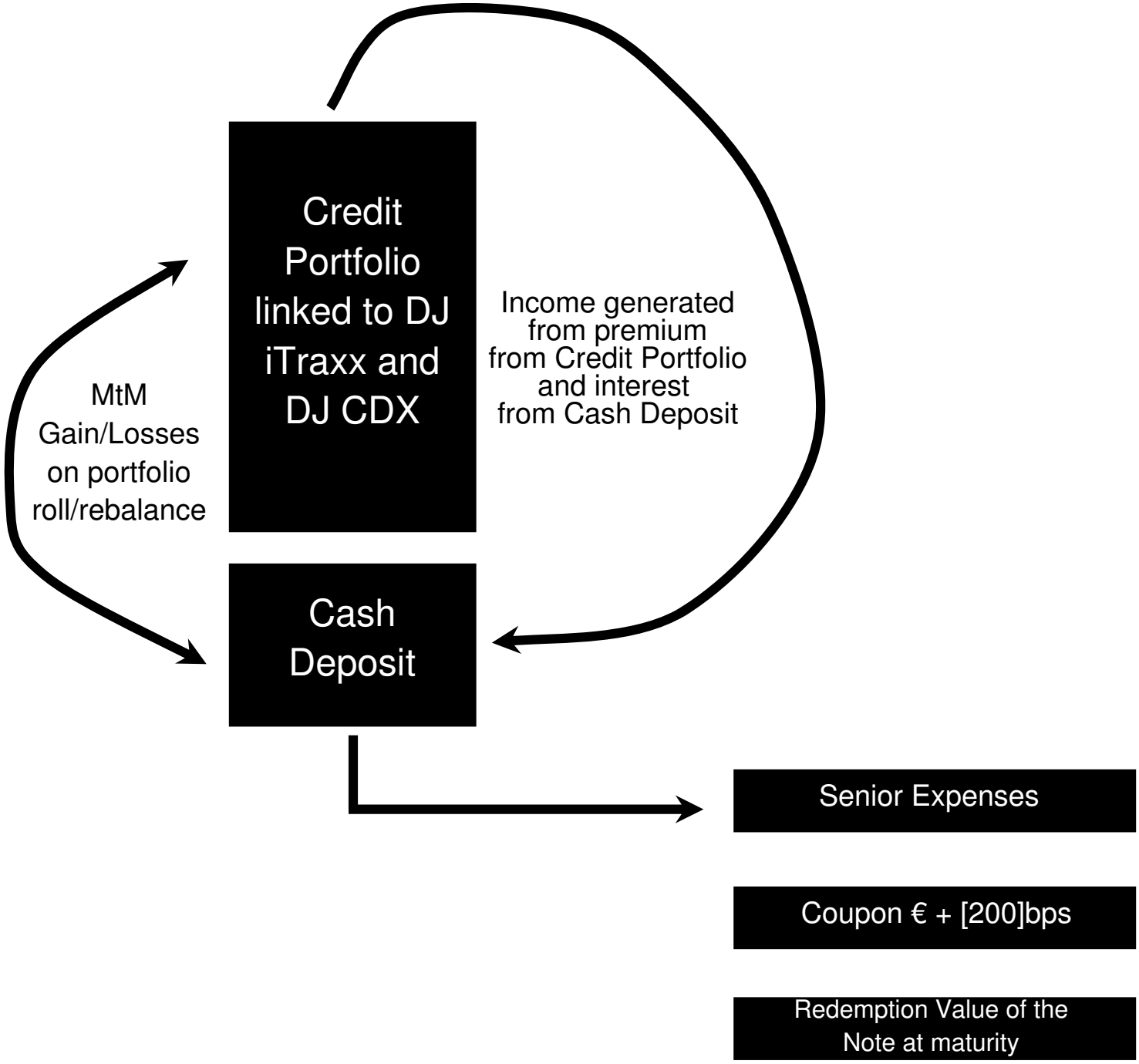
Performance

The performance of CPDOs is highly dependent on the mark-to-market (MtM) impact of changes in credit spreads. The market value of exposure is typically driven by the credit default swap (CDS) premium to be paid for protection on a portfolio of names with credit risk, or a credit index such as iTraxx or CDX.

$$\begin{aligned} \text{Net asset value (NAV)} &= \text{deposit amount} \\ &\quad + \text{MtM of the risky CDS portfolio} \end{aligned}$$

$$\begin{aligned} \text{shortfall} &= \text{PV (liabilities)} - \text{NAV} \\ &\quad + \text{cushion} \times \text{Note Notional} \end{aligned}$$

- The shortfall drives the leverage. Here, cushion is in terms of percentage of the Note Notional. It is added to give some floor value to leverage even when PV (liabilities) is close to NAV.


Credit Portfolio linked to DJ iTraxx and DJ CDX
Income generated from premium from Credit Portfolio and interest from Cash Deposit
MtM Gain/Losses on portfolio roll/rebalance
Cash Deposit
Senior Expenses
Coupon € + [200]bps
Redemption Value of the Note at maturity

Structure

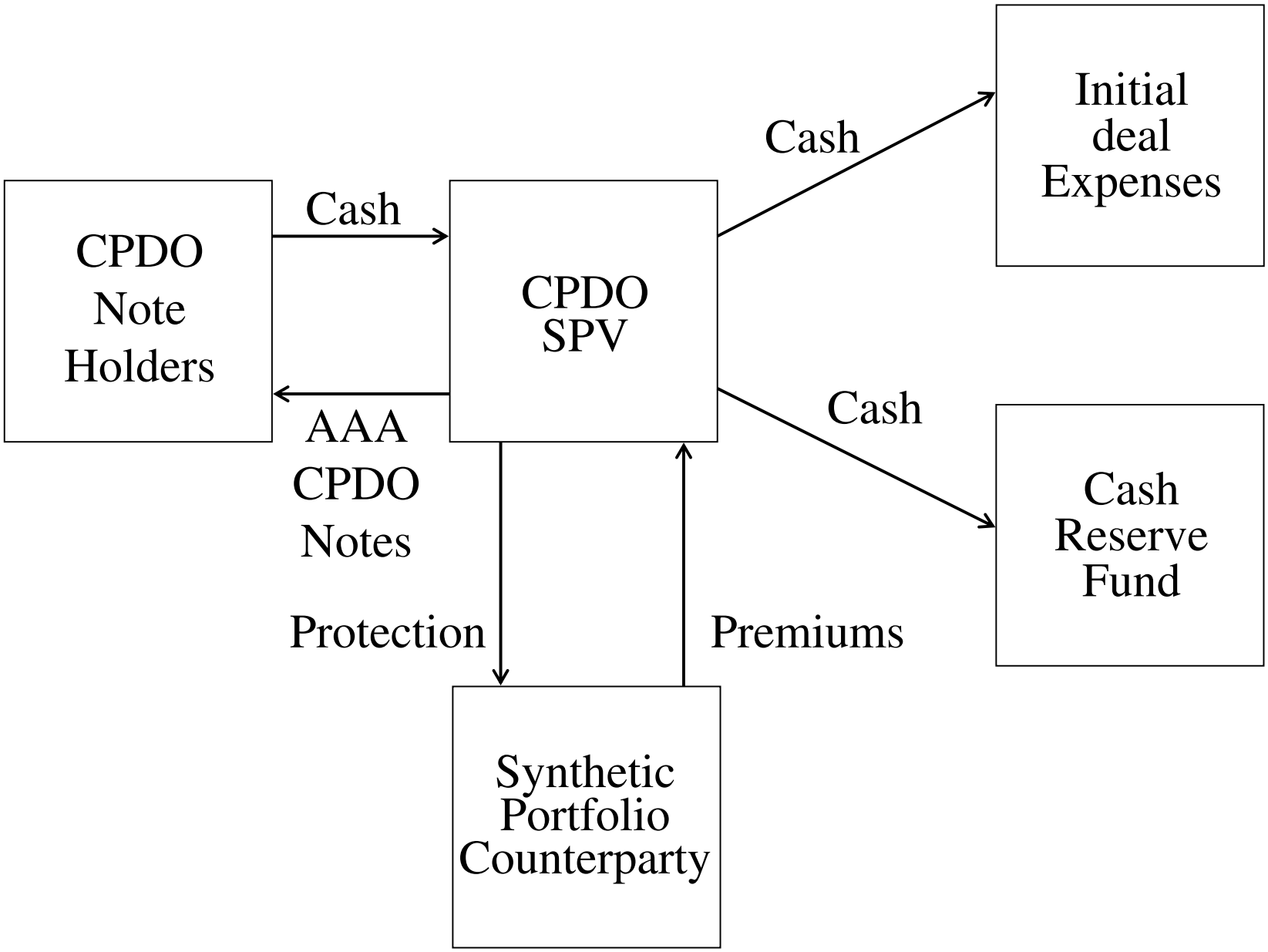
CPDO notes are issued out of a special-purpose vehicle (SPV). The proceeds from the issuance are deposited by the arranging bank in a cash account or equivalent collateral to generate regular returns, e.g. based on Libor or Euribor.

- The arranging bank enters into a credit default swap (CDS) on a risky credit portfolio, typically referencing a credit default swap index, say, CDX and iTraxx.

- The notional of this CDS is a multiple of the posted collateral, thereby creating leverage.

Actually, when the CPDO is further away from achieving its commitment on coupon payments, the more leverage it will employs (within certain limit). This is exactly the "gambler chasing losses".

Initial CPDO Flows



Initial CPDO flows

- CPDO note holders exchange cash for the CPDO's rated notes.

How to use the cash?

1. A small part is used to pay the initial deal expenses such as rating agency fees, legal expenses and any up-front sponsor fees.

2. The majority is deposited into a cash reserve account, where it is invested in very liquid and very high-credit quality assets, such as AAA-rated guaranteed investment contracts.

- CPDO's synthetic investment

  – selling protection on the investment-grade corporate CDS indexes. The CPDO's counterparty is the bank that sponsors the CPDO.

- The premium as well as any gains and losses from the CDS plus the return on the posted collateral are passed on to investors by paying interest on their investment and adjusting the collateral amount by realized gains or losses.

- During the term of the transaction, the notional of the CDS exposure is adjusted based on automatic leverage rules as the outstanding collateral amount increases or decrease.

The leverage is based on the following formula:

Min((PV of future libability + cushion − NAV)/(PV of premium × multiplier), Maximum Leverage)

Higher value of cushion chosen means higher "perceived" shortfall, and this results a higher leverage.

Assuming a PV of liabilities of 112%, a cushion of 5%, NAV of 100%, a PV of premium of 1.4% and a multiplier of 0.75, the calculation is:

$$\begin{aligned} \text{Target leverage} &= \min((112\% + 5\% - 100\%)/(1.4\% \times 0.75), 15) \\ &= \min(16, 15) = 15. \end{aligned}$$

- Leverage tends to fall when the transaction performs well (increase of the portfolio NAV) and to increase when it performs poorly (decrease of the portfolio NAV).

- Exposure to default risk is managed through a process of automatically rolling every six months out of the existing credit index (now off the run) and into the new, on-the-run credit index.

Cash-in event

- Once the current Note NAV equals the present value of the payments due under the Note, the Credit Index Portfolio will be unwound and no further credit exposure taken.

- Though this mechanism ensures that the transaction locks in the positive performance of the risky strategy, it does also cap the return of the strategy at the coupon payments. Rather than trying to maximize return, a CPDO stops taking on risk when profits are sufficient to ensure targeted rate of return.

Cash-out event

This is triggered when the Net Asset Value of the whole portfolio falls below a minimum percentage of the note notional (for instance 10%).

Triple-A rating (CPDOs pay a stated coupon at high ratings)

- The rationale for the triple-A rating is clearly the perceived low risk. The low risk is as a result of the investment grade underlying credit risk of the portfolio.

- The risk is mitigated via the rolls, which limits the risk horizon to 6 months. Rating agency stress tests obviously support triple-A probability of repayment of coupons and principal. Older series become illiquid quickly. Rolling the index swaps each 6 months ensures greater liquidity in the credit portfolio and help to keep rebalancing costs low.

The value in the CPDO relies on using a highly leveraged structure to monetise an expectation of no defaults (specifically early defaults) in the underlying credit risk assumed.

The primary credit risk mitigation is based on:

- *Diversification* – credit is diversified through selling on 250 names (equally weighted between CDX IG-7 and i-Traxx).

- *Rolling the index* – the requirement of semi-annual rolls to a new series of the underlying credit indices is designed to reduce credit risk. When the index rolls, protection is bought on the off-the-run index. New protection is sold on the current on-the-run index.

  In effect, the rolls should lead to the less liquid and deteriorating names dropping out every time the index rolls. This should reduce the risk. In effect, the investor's risk is always limited to the default of any one name in the portfolio within the relevant six month period.

CPDOs versus CDOs

- Unlike CDOs, there is a possibility of cashing-in to a risk free coupon paying bond prior to maturity, which means that credit risk need not be taken for the full life of the note.

- The CPDO has no direct price exposure to default correlation of the underlying reference obligors.

- The CPDO does not suffer from adverse portfolio selection as the credit portfolio is linked to the on-the-run credit default swap indices.

Example

1. Dealer invests the US$100m in high quality collateral.

2. Dealer sells protection on US$1,500m notional on credit indices – typically, the DJ CDX IG-7 and the iTraxx.

3. Dealer must re-balance the hedge every 6 months by rolling the credit index position to the new series of the index.

- If there are no losses, then the investor receives full return of principal invested. However, if there are credit events on the names in the index, then normal settlement is effected, lowering the principal returns to the investor.

- Assume the example above – US$100m 10-year note. The note principal is placed in high quality collateral. Assume that it earns money market returns at LIBOR flat.

- The investor is selling protection on US$1,500m on the 10 year selected credit indices. Assuming the spread on the index is 35bp pa. The investors earns 525bp $=$ 35bp $\times$15.

- Investor receives LIBOR plus 200bp pa on the CPDO.

- Assume that the arranger receives 60bp pa as management fee.

- The net position is 265bp $=$ 525 bps $-$ (200 $+$ 60)bps.

- The excess spread is set aside to cover certain losses/costs:

  1. Roll losses/costs (as index is reset)
  2. Mark-to-market losses/costs.

Summary of cash flows

Credits

- Accrued interest in the collateral account.
- Credit premiums from the risk holding (CDS index portfolio).
- Realized MTM gains on any roll or rebalancing date.

Debits

- Note coupon payments.
- Losses from defaults.
- Realized MTM losses on any roll or rebalancing date.
- Fees.

Build up of excess net spread and cash-in event

The cash-in effect inherent in the CPDO structures is predicated on the net spread. The excess net spread (after adjustment for the costs that must be covered) builds up over time if there are small number of defaults.

This creates a buffer that prior to scheduled maturity covers the remaining coupons and fees. It effectively defeases the outstanding interest owed, allowing the transaction to be cashed in before the final legal maturity.

In a typical structure, the model indicates:

- 50% probability of being cashed in after 5 years.
- 75% probability of being cashed in by 7/8 years.

The economics of the cash-in can be illustrated:

- The present value of the spread (200bp pa) is calculated to be US$15.56m.
- The net spread (assuming no defaults) is 265bp pa (US$2.65m). This equates to a cash in target of around 5.9 years

  (US$15.56/US$2.65).

In CPDOs, the notes will **default** in two cases:

- over the tenor of the deal, the NAV falls by a substantial amount and hits the cash-out trigger; typically set at 10% of the initial NAV; or
- at maturity, the NAV is not at least equal to the principal due.

Defaults in Reference Portfolio

The impact of any single default in a CPDO structure is magnified through the leverage.

- For a CPDO referencing 250 equally weighted names with leverage of 15×, a single default would lead to a reduction of the NAV by 3.6% assuming a recovery rate of 40%. Here,

$$3.6\% = 1/250 \times 100\% \times 15 \times (100\% - 40\%).$$

- In CPDOs, the default risk is often reduced through structural features. For instance, for CPDOs reference a rolling index, the exposure is taken on the "on the run" index. This means that the default risk is limited to a drop from investment grade to non-investment grade within six months.

- The structure carries significant credit risk:

  1. event risk (an investment grade credit defaulting unexpectedly);

  2. credit cycle risk (rising default rates as a result of an overall weakening in the economic and credit conditions).

- Assume 3 defaults per year over 5 years with 40% recoveries. Each default equates to 0.40% of portfolio (1/250) before recoveries. The loss after recovery is 0.24%. This is equal to US$3.6m (calculated as US$1,500 m $\times$0.24%). This means that 3 losses equal US$10.8m pa to be covered from the net spread (US$2.65m) pa.

Liquidity risk (bid-ask offer)

This is related to the required semi-annual rolls.

- The investor is fully exposed to the impact of liquidity and roll costs every 6 months when credits drop out and are replaced.
- The high leverage of the structure means that US$5bn of the CPDO issues creates US$75bn of index selling and rolls.

Spread Risk

- As spreads widen/tighten, there is the initial negative/positive mark-to-market effect on the NAV.
- Income effect: as spread widen/tighten, the structure re-contracts at a new rate, the leveraged return will also increase/decrease.

  However, higher leverage means higher exposure to credit losses.

New development since financial tsunami 2008

- When CPDOs were first issued, the credit spreads were so narrow that the products had to use maximum allowed leverage to make profits. When the credit crunch hit the markets and the spreads widened dramatically, this AAA-rated instrument lost sometimes up to 90% of its net asset value.

- On 23 May 2008, the CEO of Moody's issued a public statement: "As you may be aware, there have been reports in the news media of an error in a model that Moody's Investors Service used in certain of its ratings of European CPDOs."

Summary

- The constant proportional debt obligation is the structured product that is essentially a leveraged bet on the credit quality of a bunch of US and European investment-grade companies.

- It aims to generate income by selling protection on the two main indices of credit default swaps – the iTraxx European and the Dow Jones CDX.

- The high ratings of these deals – they have been AAA-rated investments so far – is based on two main defence mechanisms.

  1. Cash-in mechanism: The CPDO is cashed into a riskfree bond once the excess net spread income earned from CDS premium is sufficient to cover the stated coupons in the CPDOs.

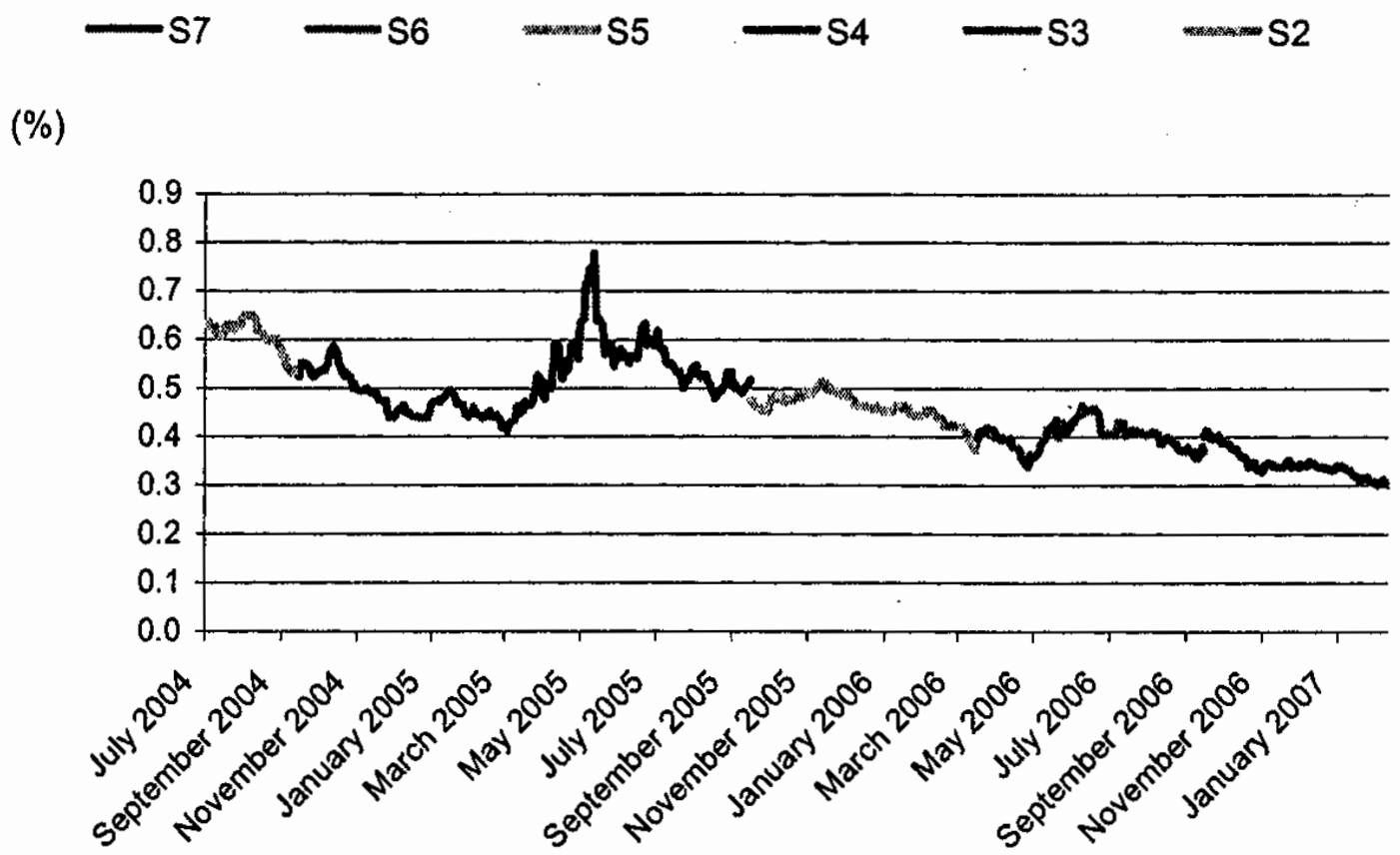
2. The exposure is rolled over with each six-monthly change in the index series, so that deteriorating credits are expunged on a regular basis.

- The high leverage involved means that the investor's principal acts as a first-loss piece on a much bigger – up to 15 times bigger – exposure to the indices.

- Liquidity crunch: when CPDOs perform their regular 6-month rolls, this would lead to a huge demand on the protection seller side of CDS on market credit indexes.

- If there are losses and the CPDO's net asset value begins to fall from its target, the leverage is increased to try to earn more at a faster rate. This has been compared with a gambler chasing losses.

Chart 3: Spreads On Dow Jones CDX (Investment-Grade) Five-Year Maturity

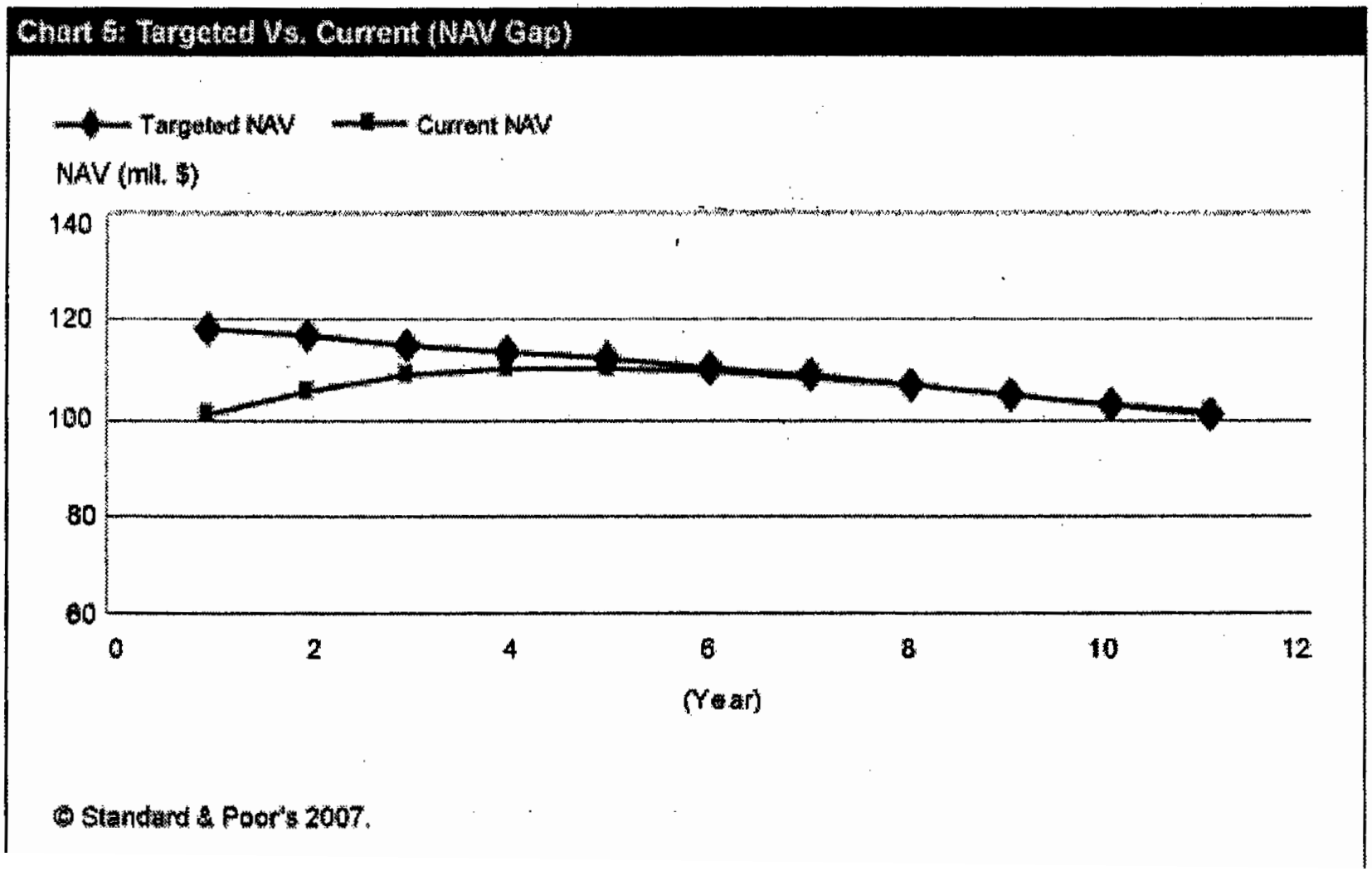


Hypothetical CPDO Transaction Features

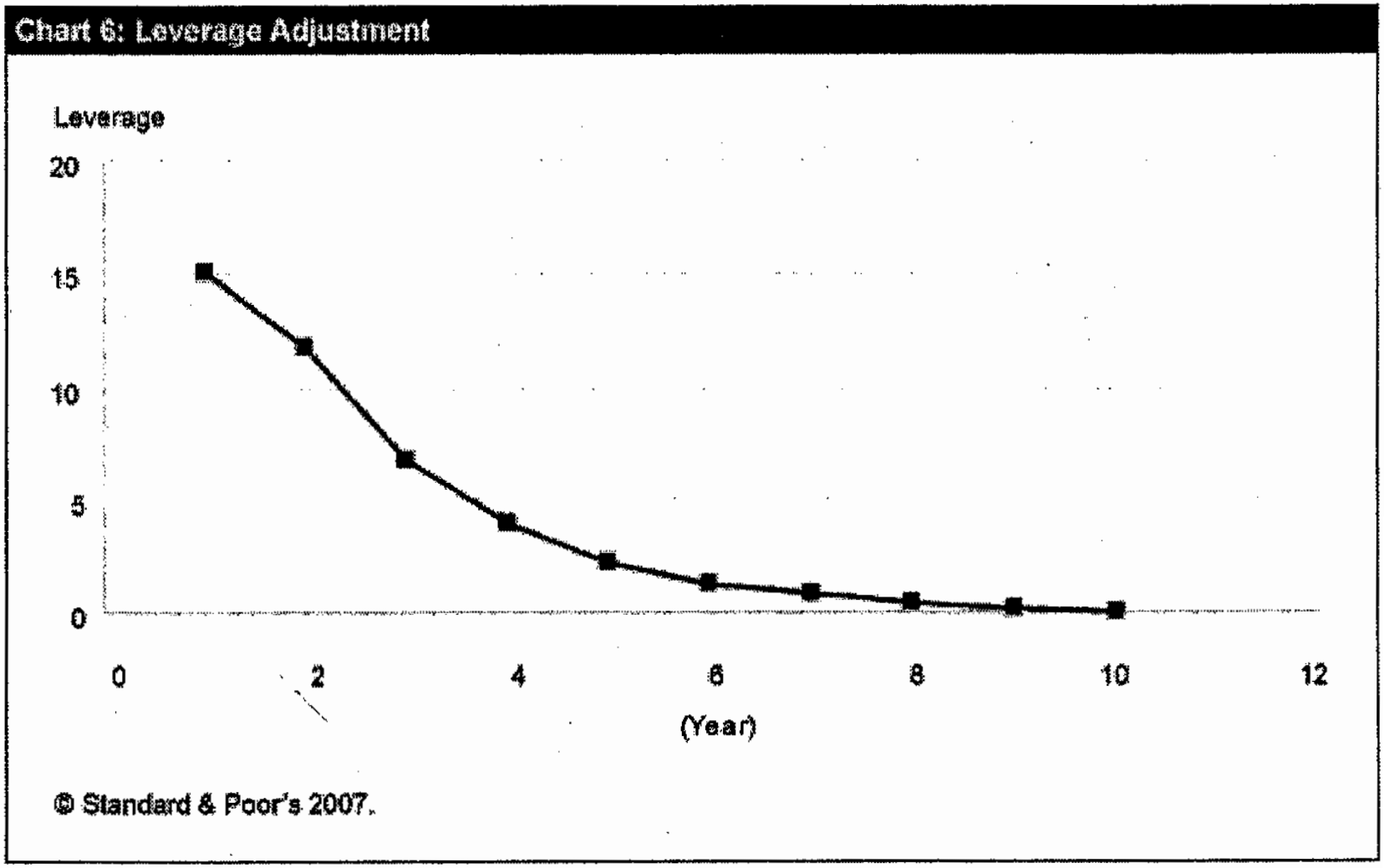
Notes issuance/liability (Mil.$)	100
Notional amount (Mil. $)	1,500
Initial leverage	15
Maximum leverage	15
Maturity (years)	10
Constant discount factor (%)	3.5
Target annual return (assuming LIBOR is 3.5% constant)	LIBOR plus 200 bps
Risky portfolio premium (bps)	40
Cash out (%)	10
Fees	None

Asset And Liabilities Mechanics Of A CPDO: An Example											
Year	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
CDS spread (bps)	40	40	40	40	40	40	40	40	40	40	40
Cash account (Mil. $)	100.00	103.50	107.12	110.87	114.75	118.77	122.93	127.23	131.68	136.29	141.06
Mark-to-market (Mil. $)	0.00	0.71	0.28	(2.20)	(6.03)	(10.76)	(16.10)	(21.81)	(27.91)	(34.35)	(41.05)
NAV (Mil. $)	100.0	104.2	107.4	108.7	108.7	108.0	106.8	105.4	103.8	101.9	***100.0***
Present value (liabilities) (Mil. $)	116.63	115.22	113.75	112.23	110.66	109.03	107.35	105.60	103.80	101.93	100.00
Shortfall	16.63	11.01	6.34	3.56	1.93	1.02	0.52	0.19	0.02	***(0.01)***	—
Risky portfolio (Mil. $)	1,500.00	1,218.70	727.20	421.90	237.50	129.50	83.82	41.59	8.05	0.00	—
Risky return (Mil. $)	1,506.00	1,223.60	730.10	423.60	238.40	130.00	84.15	41.75	8.08	0.00	—
Leverage adjustment	15.00	11.70	6.77	3.88	2.18	1.20	0.78	0.39	0.08	0.00	—
Cash in	No	No	No	No	No	No	No	No	No	***Yes***	***Yes***
Cash out	No	No	No	No	No	No	No	No	No	No	No

- The leverage starts at 15 when the shortfall is 16.63.
- "Cash-in" occurs at the end of the 9^{th} year when the shortfall vanishes.



- When the CPDO does not suffer too much default losses through its leveraged exposure to credit indices, the gap between the target NAV and actual NAV narrows as time proceeds.



- The CPDO can take a lower leverage when the excess net spread builds up over time if there are small number of defaults.

What is a Credit CPPI (constant proportional portfolio insurance)?

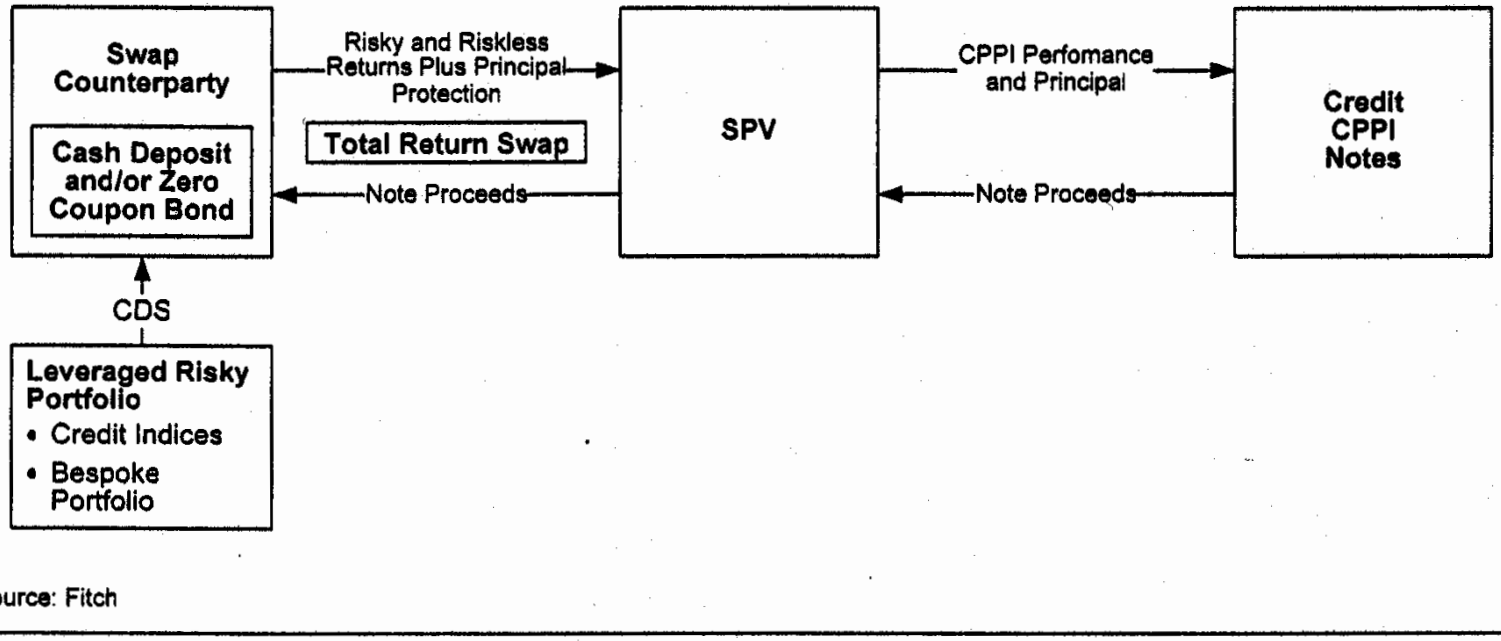
- Principal is protected by a low-risk portfolio consisting of zero coupon bonds or a cash deposit.
- Return is increased by leveraging the exposure to a risky portfolio of credit default swap (CDS) names.
- Structured to ensure that investors' principal will be returned at maturity.

— Limited downside owing to the principal protection and attractive upside; flexibility to manage exposures.

[In traditional credit products, the upside is limited to the coupon and the downside could be the entire loss of principal.]

— First issue in late 2003.

Diagram 2: A CPPI Structure with an SPV as Issuer



Source: Fitch

How does a Credit CPPI work?

- Arranger defines a bond floor
  - the zero-coupon bond is purchased on the issue date
  - price of a zero-coupon bond required to ensure note repayment at maturity [proceeds from the issuance are placed in a cash deposit account with LIBOR overnight rate of return].
- Exposure to the risky portfolio is taken synthetically
  - The combined value of the risky portfolio and the "riskless" portfolio is not permitted to fall below the bond floor.

Reserve

- Purchased zero-coupon bond
  - reserve is initially the remaining cash deposit itself
- Monitored zero-coupon bond
  - reserve is initially the difference between the value of the cash deposit and the bond floor.
- The reserve changes with any mark-to-market (MtM) gains or losses and credit losses during the life of the transaction plus the earned CDS premiums and return on the cash deposit.
- The reserve is leveraged to take a synthetic position in the risk portfolio of CDS names via the use of a constant/dynamic multiplier. The multiplier corresponds to the target leverage in the transaction.

Target Notional (Corresponds to the desired amount of risky exposure)

$$\text{target notional} = \text{reserve} \times \text{target leverage}$$

This is the amount synthetically invested in the risky portfolio at the start and each time a rebalancing occurs during the life of the transaction.

- Effective leverage = current risky exposure/current reserve

– Monitored daily against the target leverage but is allowed to fluctuate within a certain band to avoid overly frequent rebalancing.

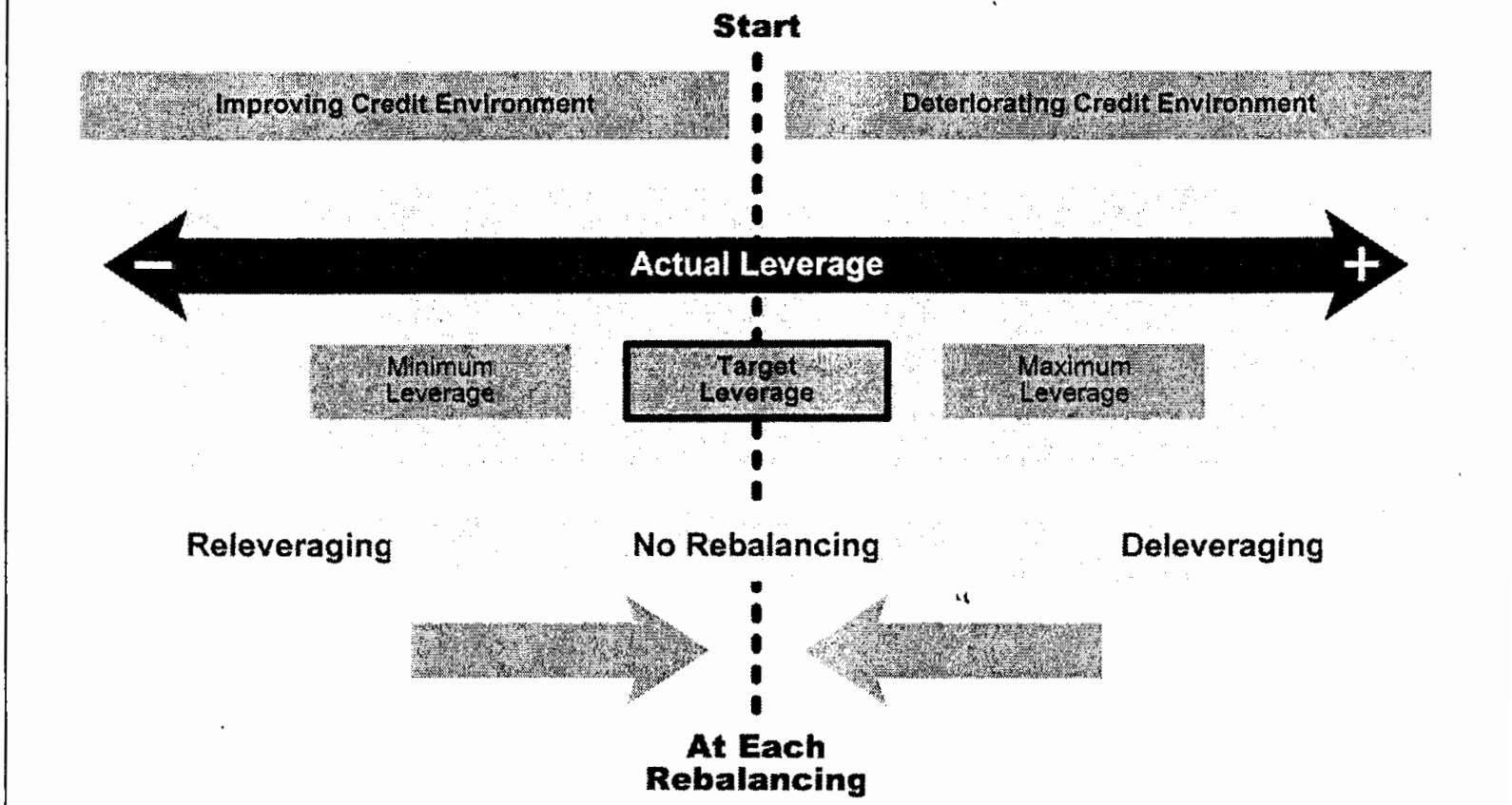
Rebalancing

- If the transaction performs well (spread-tightening), the MtM profits will lead to an increase in the value of the reserve and hence a decrease in the effective leverage. Exposure to the risky portfolio will be increased (releveraging) until the target leverage is reached.

- If the transaction perform poorly, MtM losses will cause the reserve to decrease, thereby increasing effective leverage. Exposure to the risky portfolio will be reduced (deleveraging) until the target leverage is reached.

- If the current leverage is within the minimum and the maximum levels, no rebalancing will occur.

Diagram 1: Schema of Rebalancing Process



Source: Fitch

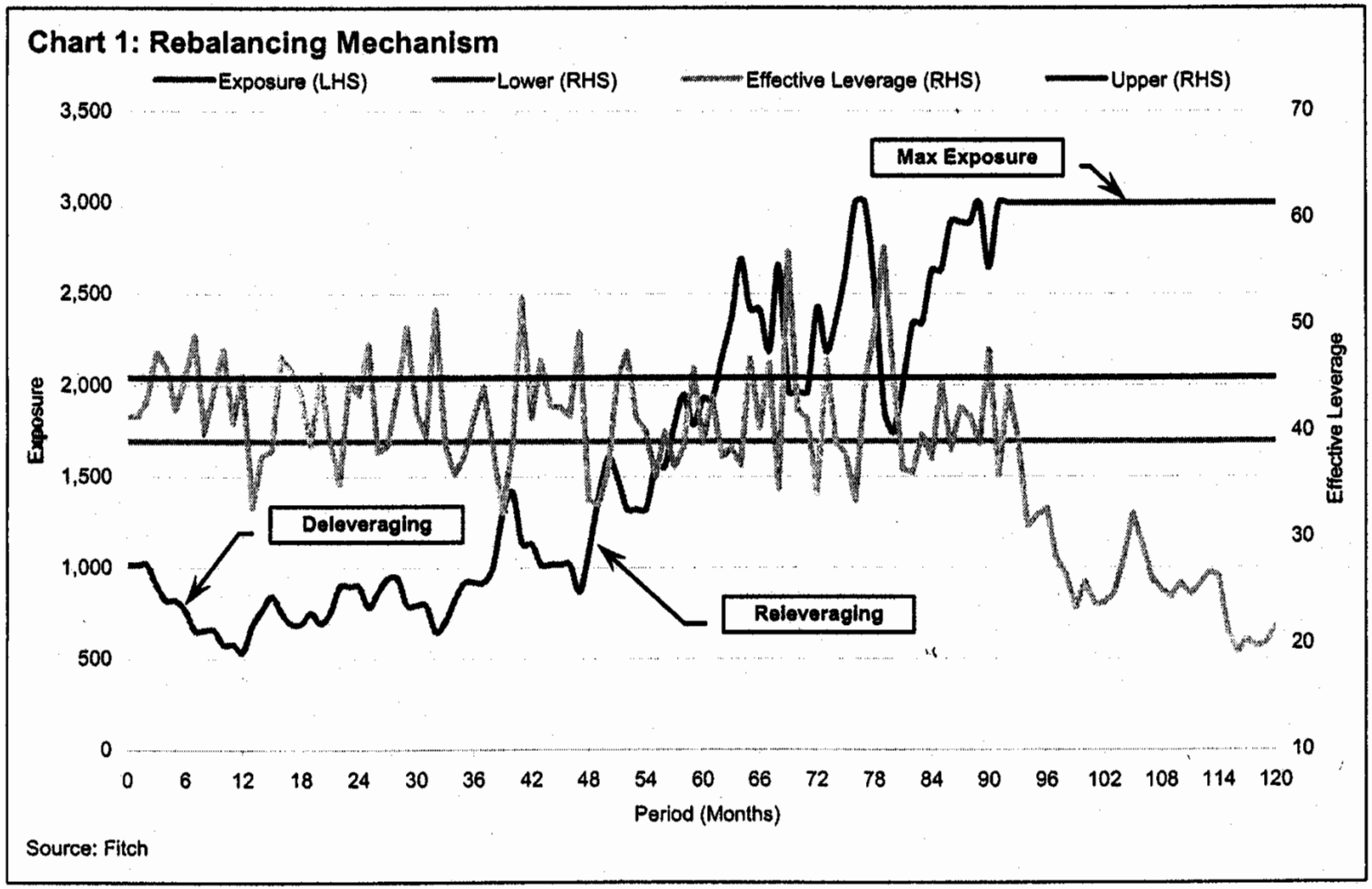


Chart 1: Rebalancing Mechanism
Exposure (LHS)
Lower (RHS)
Effective Leverage (RHS)
Upper (RHS)
Max Exposure
Deleveraging
Releveraging
Exposure
Effective Leverage
Period (Months)
3,500
3,000
2,500
2,000
1,500
1,000
500
0
70
60
50
40
30
20
10
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102 108 114 120
Source: Fitch

Cash-out event

The cash-out event occurs when the reserve falls below a minimum percentage of the notes notional (say, 5%).

- The entire risky exposure is unwounded and the transaction is wholly invested in a zero-coupon bond. Investors will only receive principal back at maturity – no more upside and coupon.

Gap risk

Market conditions could deteriorate too quickly or defaults occur too suddenly for the cash-out event to take place.

  - Fall in the value of risky portfolio exceeds the value of the reserve.

Summary

Credit CPPI seeks to maximize returns while providing full or partial principal protection.

- The subportfolio of risky exposures provides upside potential while a constant reference to a "bond floor" provides a notional put option.

– As the portfolio value increases, more of the portfolio is shifted to the "riskfree" subportfolio, providing principal protection.

- Opposite leverage mechanism as that of the CPDO.

- No cash-in event, while CPDOs cash in when the NAV (portfolio) $>$ NPV of future liabilities

- Unlimited returns, unlike CPDO whose returns are capped at coupon payments.