MAFS5250 – Computational Methods for Pricing Structured Products

Course objective

This course introduces the computational techniques in pricing of structured products. We discuss the three common classes of numerical methods: lattice tree methods, finite difference methods, and Monte Carlo simulation methods. These computational techniques are then applied to pricing structured products that are commonly traded in the financial markets. The types of options and structured products include: lookback options, barrier options, Asian options, accumulators, participating life policies and convertible bonds. Calibration of the implied lattice trees for stock prices and interest rates to traded option prices and bond prices will be considered.

Instructor

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Textbooks/ references

- Yue Kuen KWOK, *Mathematical Models of Financial Derivatives*, second edition, Springer Verlag, 2008 (Chapter 6).
- Domingo TAVELLA and Curt RANDALL, *Pricing Financial Instruments*, John Wiley & Sons, Inc., 2000.
- Rudiger SEYDEL, *Tools for computational finance*, second edition, Springer Verlag (2003).
- Claudio ALBANESE and Giuseppe CAMPOLIETI, *Advanced derivatives pricing and risk management*, Academic Press, 2006.
- Daniel J. DUFFY, *Finite difference methods in financial engineering*, John Wiley and Sons, 2006.
- Gianluca FUSAI and Andrea RONCORONI, *Implementing Models in Quantitative Finance: Methods and Cases*, Springer, 2008.
- Ali HIRSA, *Computational Methods in Finance*, Chapman and Hall, 2012.

Course content

1. Lattice tree methods
    1.1 Binomial option pricing models
    - Risk neutral valuation principle
    - Continuous limit of the binomial model
    - Multi-period extension
    - Dynamic programming procedure
    - Discrete dividend models
    - Pricing of lookback options: Hull-White scheme and Cheuk-Vorst algorithms

    1.2 Trinomial schemes
    - Discounted expectation approach
    - Multistate extension – Ritchken-Kamrad's approach

    1.3 Forward shooting grid algorithms (strongly path dependent options)

- Cumulative Parisian feature
- Call options with strike reset feature
- Floating strike arithmetic averaging call
- Alpha-quantile options
- Accumulators

2. Implied binomial trees and calibration of interest rate trees
    - 2.1 Implied binomial trees
        - Arrow-Debreu prices and structures of the implied binomial trees
        - Derman-Kani algorithm
    - 2.2 Hull-White interest rate model and pricing of interest rate derivatives
        - Analytic procedure of fitting the initial term structures of bond prices
        - Calibration of interest rate tress against market discount curves

3. Finite difference methods
    - 3.1 Discretization of the Black-Scholes equation
        - Explicit schemes: Domain of dependence, incorporation of boundary conditions; skew computational domain
        - Crank-Nicolson scheme: Thomas algorithm
        - Projected successive-over-relaxation method
    - 3.2 Numerical approximation of auxiliary conditions
        - Initial conditions and terminal payoffs
        - Lookback options – Neumann boundary conditions
        - Barrier type options
    - 3.3 Properties of numerical solutions
        - Truncation errors and order of convergence
        - Numerical stability
        - Spurious oscillations

4. Exotic structured products
    - 4.1 Participating life insurance policies
        - Product nature: bonus distribution mechanism
        - Finite difference scheme
        - Numerical results and pricing behavior
    - 4.2 Convertible bonds
        - Embedded features
        - Modeling considerations in convertible bond pricing models
        - Lattice tree algorithm (ad-hoc approach)
        - Finite difference scheme
        - Analysis of pricing behavior

5. Monte Carlo simulation
    - 5.1 General formulation of Monte Carlo procedure
        - Expected value and variance of estimate
        - Multistate extension – correlated random samples

- Computational efficiencies
- Calculating the greeks

5.2 Variance reduction techniques

- Antithetic variates method
- Control variate method
- Importance sampling method

5.3 Valuation of American options

- Method of parametrization of the early exercise boundary
- Linear regression method via basis functions

6. Advanced numerical schemes for pricing path dependent options

6.1 Discretely sampled fixed strike Asian options

- Chance of numeraire approach
- Choice of Markovian state variable
- Construction of the Crank-Nicolson scheme

6.2 Discretely sampled lookback options

- Fixed strike lookback call option
- Floating strike lookback option

Grading policy

Two computer assignments (work as a group of 2)	30%
• Reverse convertibles	
• Participating policies	
90-minute first test	30%
120-minute final examination	40%
	Total = 100%