2.3 Counterparty risks of swaps

Credit risk of defaultable currency swaps

- A financial swap is the exchange of cashflows based on an underlying index under some prescribed terms.
- Two major sources of risks
  - rate risk (change in interest rate or exchange rate)
  - credit risk (either party may default)

  The swap default risk is two-sided.
- Maximum loss associated with the credit risk is measured by the swap's replacement cost.

Question How much spread is appropriate to cover the swap credit risk of the swap counterparty?

Reference

H.Yu and Y.K. Kwok, "Contingent claim approach for analyzing the credit risk of defaultable currency swaps," AMS/IP Studies in Advanced Mathematics, vol. 26, p.79-92 (2002).

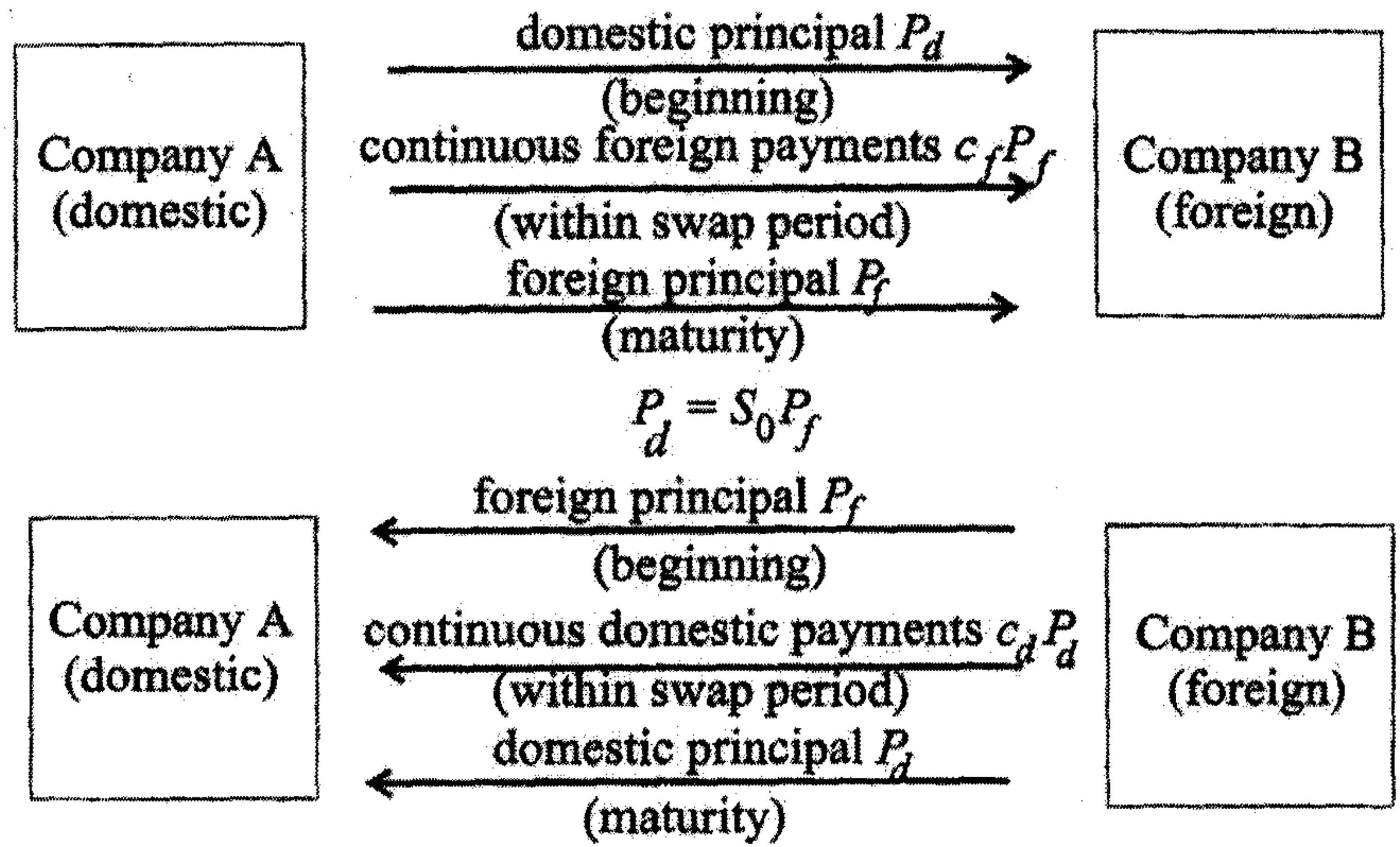
Considerations in credit risk analysis

The settlement payment to the swap counterparty upon inter-temporal default depends on the settlement clauses in the swap contract.

- Under the full (limited) two-way payment clause, the non-defaulting counterparty is required (not required) to pay if the final net amount is favorable to the defaulting party.
- When the swap is favorable to the non-defaulting party, it receives only the fraction $1 - w$ of the market quotation value of the swap agreement.

Remark

It is too simplified to estimate the spreads on the higher-rated and lower-rated swaps from the quality spread on bonds of similar credit categories.



Cashflows between the two currency swap counterparties, assuming no intertemporal default.

Cashflows between currency swap counterparties

- Domestic company A has a comparative advantage in borrowing domestic loan but it wants to raise foreign capital (reverse situation for its counterparty B).
- For simplicity of analysis, we assume that the exchange payments are continuous.
- The swap rates are chosen such that the value of the swap contract is set to be zero at initiation.
- When the firm value F of company A falls to the threshold level H, company A is forced to reorganize.
- Under the risk neutral measure Q, the dynamics of the exchange rate S and firm value F of company A are governed by

$$\begin{aligned}\frac{dS}{S} &= (r_d - r_f)\,dt + \sigma_S\,dZ_S\\ \frac{dF}{F} &= r_d\,dt + \sigma_F\,dZ_F\end{aligned}$$

  where r_d and r_f are the domestic and foreign riskfree interest rates and $dZ_S\,dZ_F = \rho\,dt$.

We stay in the domestic currency world.

$$V(S,t) = \text{value at time } t \text{ of the riskfree currency swap to company } B$$
$$\overline{V}(S,F,t) = \text{value at time } t \text{ of the defaultable swap to company } B$$

Governing equations

(i)
$$\frac{\partial V}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + (r_d - r_f)\frac{\partial V}{\partial S} + (P_f c_f S - P_d c_d) - r_d V = 0,$$
$$0 < S < \infty, t > 0,$$

with terminal payoff

$$V(S,T) = P_f S - P_d.$$

(ii)
$$\frac{\partial \overline{V}}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 \overline{V}}{\partial S^2} + \rho\sigma_S\sigma_F SF \frac{\partial^2 \overline{V}}{\partial S \partial F} + \frac{\sigma_F^2}{2} F^2 \frac{\partial^2 \overline{V}}{\partial F^2},$$
$$+[r_d F - (P_f c_f S - P_d c_d)]\frac{\partial \overline{V}}{\partial F} + (r_d - r_f) S \frac{\partial \overline{V}}{\partial S} + (P_f c_f S - P_d c_d) - r_d V = 0,$$
$$0 < S < \infty, \quad H < F < \infty, t > 0.$$

Prescription of auxiliary conditions

1. Limited two-way settlement

- $\overline{V}(S,F,T) = \begin{cases} P_f S - P_d, & F > H \\ (1-w)\max(P_f S - P_d, 0), & F \leq H \end{cases}.$
- $\lim_{F\to\infty} \overline{V}(S,F,t) = V(S,t)$ for all t.
- $\overline{V}(S,H,t) = (1-w)\max(V(S,t),0)$
- When $S = 0$, it will stay at that level for all later times. The foreign payments become worthless, and the swap contract behaves like a bond where B pays the continuous payments $c_d P_d$ and final par value P_d.

  Present value of the sum of these cashflows

  $$= P_d\left\{e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}]\right\}.$$

  $$\overline{V}(0,F,t) = -P_d\left\{e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}]\right\}\mathbf{1}_{\{F>H\}}.$$

2. Full two-way settlement

   B has to honor the swap contract even when A becomes default.

   - $$\overline{V}(S,F,T) = \begin{cases} P_f S - P_d & F > H \\ P_f S - P_d & F \leq H \text{ and } P_f S - P_d \leq 0 \\ (1-w)(P_f S - P_d) & F \leq H \text{ and } P_f S - P_d > 0 \end{cases} .$$
   - $$\overline{V}(S,H,t) = \begin{cases} (1-w)V(S,t) & V(S,t) > 0 \\ V(S,t) & V(S,t) \leq 0 \end{cases} .$$
   - $$\overline{V}(0,F,t) = -P_d \left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\}.$$

Solution approach

First c_d is fixed, we then find c_f such that the swap value at initiation is zero.

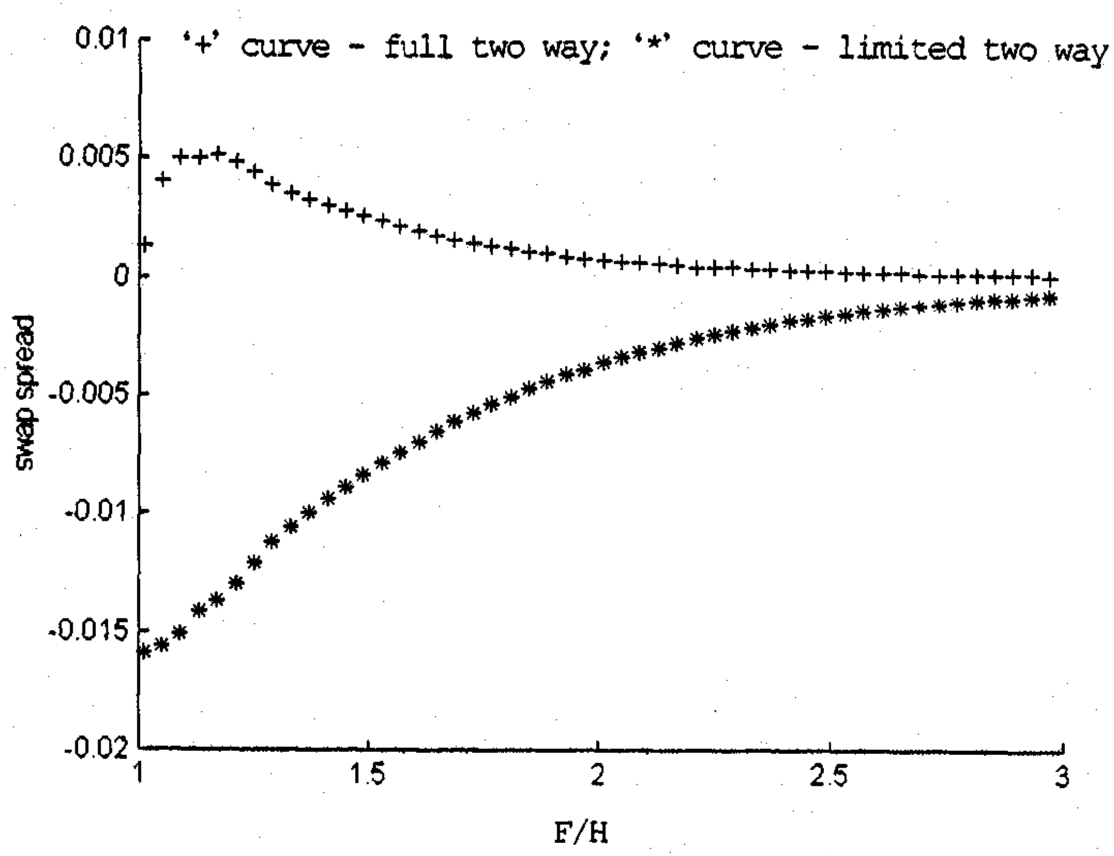


Figure 2. The relationship between the swap spread on c_f and the firm credit rating as measured by F/H, where F is the firm value and H is the default level. The parameter values are $P_d = 1, S_0 = 2, S = 2, c_d = 8\%, T = 4, r_d = r_f = 6\%, \sigma_S = 15\%, \sigma_F = 25\%, w = 0.75$ and $\rho_{SF} = 0.25$.

- The swap contract is in-the-money to the defaultable party (based on the given set of parameters).

Observations

1. The swap spreads are negative for the limited two-way payment clause. This is expected as the limited settlement clause gives advantage to party B since B can be excused from honoring an out-of-the-money swap contract to itself when A defaults. The swap spread narrows as the credit rating of A improves, and it becomes essentially zero as the ratio F/H goes beyond 3.

2. For the full two-way settlement, the swap spread curve reveals that the swap spreads are always positive and the spread achieves a maximum value at certain levels of F/H. The positivity of the swap spreads reflects the replacement cost to the non-defaulting party since it receives only a fraction of the market quotation of the swap contract upon default of the counterparty.

3. Since the swap contract is in-the-money to the defaulting party A, the loss to counterparty B is zero when A defaults. Hence, the swap spread value becomes zero when F hits H.

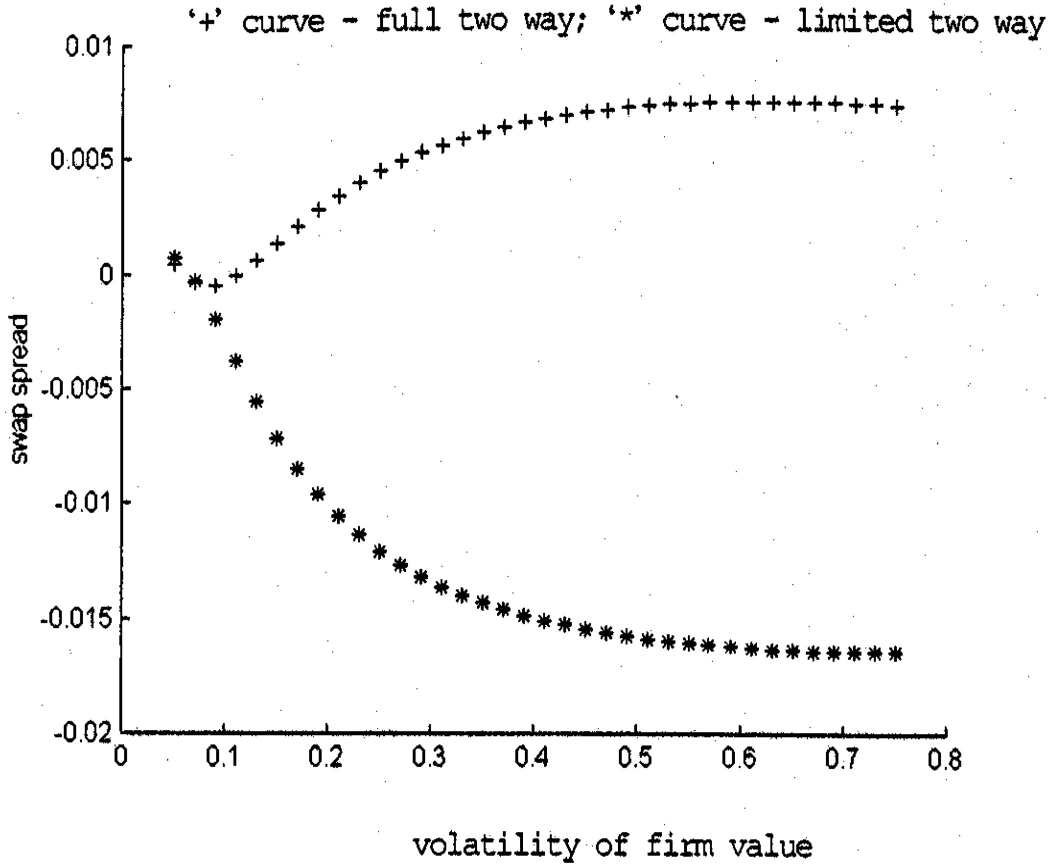


Figure 3. The relationship between the swap spread on c_f and the volatility of the firm value, σ_F. The same set of parameter values are used as those in Figure 2, except that σ_F is allowed to vary and $H = 100, F = 125$.

- When the firm value becomes more volatile, the possibility of default increases and the swap spreads become widened, that is, more negative for the limited two-way payment and more positive for the full two-way payment. The swap spreads tend to some asymptotic values at high σ_F.

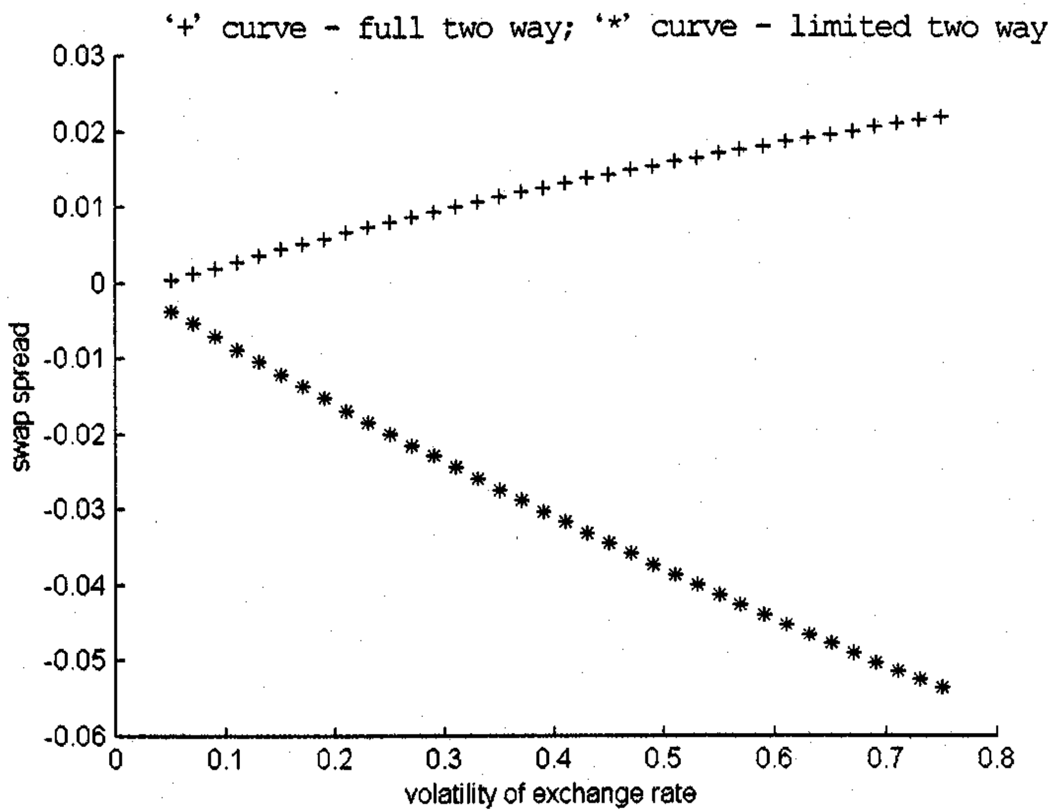


Figure 4. The relationship between the swap spread on c_f and the volatility of the firm value and exchange rate being positive, the firm has a higher possibility to default when the exchange rate becomes more volatile. Therefore, the value of swap spread increases in magnitude at higher value of σ_F for both full and limited two-way settlement clauses.

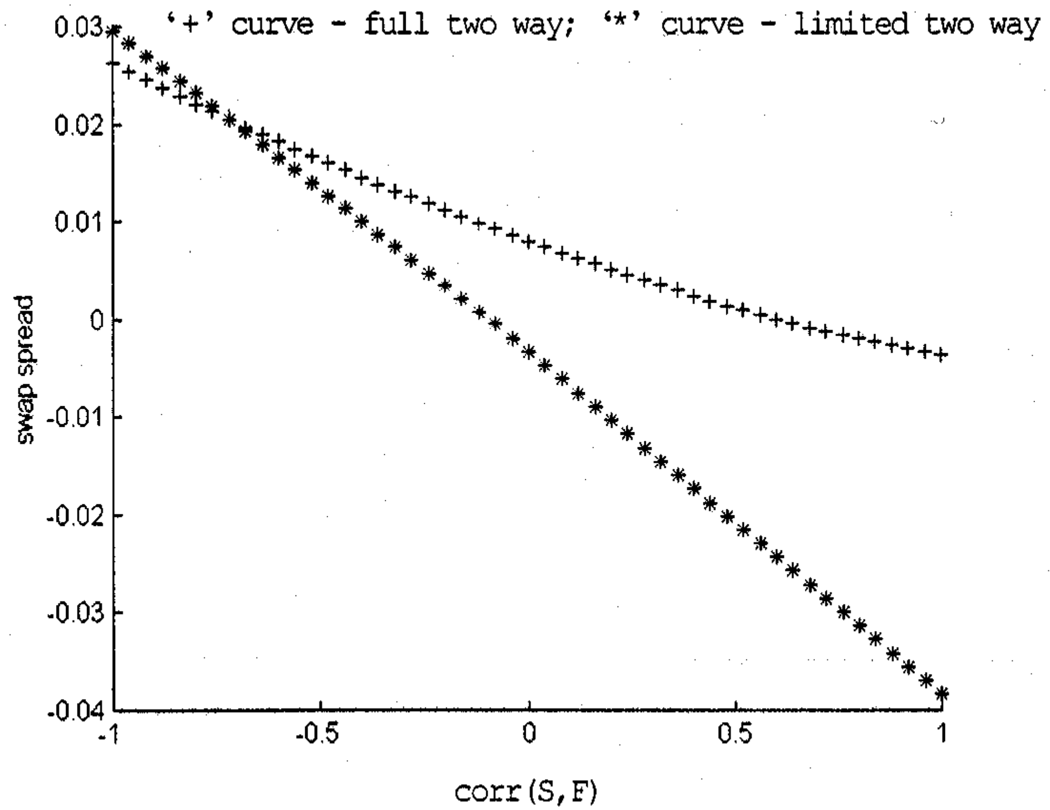


Figure 5. The relationship between the swap spread on c_f and the correlation coefficient between exchange rate and firm value, ρ_{SF}.

- For both full and limited two-way settlement clauses, the swap spreads are decreasing functions of the correlation coefficient.

Explanations

1. For full two-way settlement, the swap spread is always positive since B always loses upon the default of the counterparty. Suppose the correlation is highly positive and S increases, there is a higher tendency for the increase of F (that is, less susceptible to default) so the expected loss to B associated with the replacement cost drops.

2. On the other hand, when S decreases, the swap becomes more in-the-money to A. Correspondingly, the expected replacement cost incurs to B when A becomes default tends to a small value. Both arguments explain why the swap spread decreases when the correlation coefficient increases.

3. For limited two-way settlement, the above argument for the drop in swap spread with more positive correlation still applies, except that the swap spread can decrease beyond the zero value at high positive correlation.