Double barrier options

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A double barrier option has two barriers, one above and the other below the current stock price. It is considered as a path dependent option since the payoff to the holder depends on the breaching behaviors of the stock price process with respect to the two barriers. The double barrier option contract specifies three different payoffs, which are dependent on whether the up-barrier or down-barrier is hit, or no breaching of either barrier throughout the whole life of the option. A barrier is said to be of knock-out type if the resulting payoff upon hitting by the stock price is a rebate payment (the rebate amount may depend on the time of hitting), and knock-in type if the holder receives a new option upon hitting. The barrier feature may be applied over the whole life or a partial life of the option. A great number of double barrier options can be designed to achieve a wide variety of risk management functions through various structures. Like single barrier options, an investor buying a double barrier option may use the more exotic forms of the double barrier feature to achieve reduction in option premium, match investor's belief about the future movement of the stock price process and/or match his specific hedging needs more.

Let the random time variables τ_{up} and τ_{down} denote the first passage times that the stock price hits the up-barrier and down-barrier, respectively, and T be the maturity date of the option. We may classify double barrier options into different classes according to the payoff patterns that depend on the relative order of the times τ_{up}, τ_{down} and T.

1. $\tau_{up} < \min(\tau_{down}, T)$.

This corresponds to the scenario where the up-barrier is hit before the down-barrier within the life of the option. For example, with an *up-barrier knock-in double barrier option*, the holder receives a new option if within the option life the up-barrier is breached before the down-barrier is breached, and the option expires worthless if otherwise.

2. $\tau_{up} < \tau_{down} < T$.

Within the option's life, the breaching of the up-barrier occurs before that of the down-barrier. One example is the *sequential double barrier option* where the option is knocked out if the up-barrier and the down-barrier are breached sequentially. Equivalently, one may visualize that the sequential double barrier option becomes a down-and-out single barrier option when the up-barrier is breached.

3. $\min(\tau_{up},\tau_{down}) < T$.

Prior to maturity, either the up-barrier or the down-barrier is breached. In a *one-touch knock-out double barrier option*, the option is knocked out if at least one of the barriers is breached during the life of the option. Rebate payment may be paid upon the knocking out of the option.

4. $\max(\tau_{up}, \tau_{down}) < T$.

Both the up- and down-barriers are breached within the option's life. The *double-touch knock-out double barrier option* is knocked out if and only if both barriers have been breached before the maturity date of the option.

One can see that the combination of an up-barrier knock-in option and an up-barrier knock-out option is equivalent to the standard European option. Similarly, the one-touch knock-in option can be decomposed into an up-barrier knock-in options and a down-barrier knock-in option.

More innovative types of payoffs that are dependent on the order of breaching can be structured. One example is the *double up-and-in call option*. If none of the barriers is breached throughout the life of the option, then the option expires worthless. Suppose the stock price breaches the up-barrier before the down-barrier, the option becomes a vanilla European call. If the stock price reaches the down-barrier before the up-barrier, the option becomes an up-and-in call option with a new up-barrier and a new strike price. That is, when the stock price hits a lower threshold, the option is converted into a similar one but with different parameters.

In an occupation time derivative with double barriers, the derivative payoff depends on the amount of occupation time that the stock price stays within certain range or corridor. In 1994, Societe Generale introduced the option termed BOOST (Banking On Overall STability). The option is characterized by a corridor $[a,b]$ for the stock price process. Whenever one of the barriers is breached, the option terminates and the holder receives a payoff that is proportional to the total occupation time that the stock price stays within the corridor since initiation.

The pricing of a double barrier option amounts to the valuation of the probability density that the stock price stays within the range bounded by the up-barrier and down-barrier, and the density functions of the first passage times that the stock price breaches the barriers. The price of a double barrier option depends on the following major factors: (i) the relative position of the two barriers with respect to the current stock price, (ii) the barrier types, either knock-in or knock-out, (iii) the order of activation of the barriers, (iv) the payoff functions upon hitting the barriers, (v) the monitoring frequency of the barriers. Most of the valuation formulas derived in the literature assume continuous monitoring of the barrier and lognormal distribution of the stock price process. The price formulas and pricing methods for most types of double barrier options can be found in the articles / book cited below.

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