Enhanced Equity-Credit Modeling for Contingent Convertibles

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Abstract

Contingent convertibles are characterized by forced equity conversion under accounting trigger, which occurs when the capital ratio of the issuing bank falls below some contractual threshold. Also, under the point-of-non-viability trigger, the supervisory authority may enforce equity conversion when the financial health of the bank deteriorates to the distressed level. In this paper, we propose an equity-credit modeling of the joint process of the stock price and capital ratio that integrates both the structural approach of accounting trigger and reduced form approach of point-of-non-viability trigger of equity conversion. We also construct effective Fortet algorithms and finite difference schemes for numerical pricing of CoCo bonds under various forms of equity conversion payoff. The pricing properties of the CoCo bonds under various contractual specifications and market conditions are examined.

Keywords: Contingent convertibles, equity-credit modeling, Fortet algorithms

1 Introduction

The contingent convertible bond (CoCo) provides a higher coupon rate compared to its non-convertible counterpart since it is embedded with a loss absorption mechanism that is triggered when the capital of the issuing bank falls close to the regulatory level as required by the Basel Committee on Banking Supervision (BCBS). At a triggering event, the bond is automatically converted into equity of the issuing bank (or an equivalent amount of cash). The equity conversion is meant to provide fresh capital to the issuing bank when it is under financial distress. As a result, this can help to mitigate the chance of a systemic banking crisis, thus minimizing the use of taxpayers’ money to bail out distressed financial institutions.

Since the first issuance of the Enhanced Capital Notes by the Lloyds Banking Group in December 2009, there has been an active discussion on the triggering mechanism and loss absorption design of CoCo bonds. In a typical contractual design of a CoCo bond, there are two possible triggering mechanisms: core tier-1 accounting trigger and regulatory trigger. In an accounting trigger, the capital ratio is chosen as the indicator on a bank’s financial health. For example, when the capital ratio falls below a predetermined level, the bond is converted automatically into equity in order to boost the bank’s capitalization. In a regulatory trigger, sometimes called the point-of-non-viability
(PONV) trigger, the banking supervisory authority holds the discretion to decide whether a bank is insolvent or not, and determine accordingly whether equity conversion should be activated. As noted in a recent survey on the contractual features of CoCo bonds (Avdjiev et al., 2013), most of the CoCo issuers have chosen the accounting trigger as the de-facto mechanism.

There are a number of papers that address pricing and risk management of a CoCo bond using various modeling approaches. The structural approach starts with the modeling of a bank’s balance sheet dynamics that allows one to analyze the impact of the issuance of contingent convertibles on the capital structure (Albul et al., 2010; Brigo et al., 2015; Glasserman and Nouri, 2012; Pennacchi, 2011). Cheridito and Xu (2015) apply the reduced form approach, commonly used for pricing of credit derivatives, to price CoCo bonds. The reduced form approach has better flexibility to perform calibration of model parameters using the market prices of traded derivatives, like the credit default swap (CDS) spreads. Also, Spiegeleer and Schoutens (2012) propose an easy-to-use equity derivative approach by approximating the accounting trigger of the capital ratio by the first passage time of the stock price process to an implied barrier level. Following a similar approach, Corcuera et al. (2013) pursue a smile conforming model that assumes the stock price to follow a Levy process. Gupta et al. (2013) discuss pricing issues related to various contractual features of CoCo bonds and resort to numerical methods for the pricing of CoCo bonds with a mean-reverting capital ratio. Wilkens and Bethke (2014) report the empirical assessment of aforementioned approaches and find that the equity derivative approach implies a hedging ratio that may be practically useful with reference to the risk management of CoCo bonds during the sample period of 2011. Leung and Kwok (2015) model the regulatory trigger using the Parisian feature where equity conversion is activated when the capital ratio stays under the non-viable state cumulatively over a certain period of time.

A proper modeling of the capital ratio is crucial with regard to the contractual design of a CoCo bond with accounting ratio trigger. Since the capital ratio is a balance sheet quantity, the structural modeling approach provides a natural starting point for pricing CoCos. The structural approach, however, does not usually possess the flexibility for calibration to traded security prices and fails to generate a reasonable shape of the credit spread. Furthermore, as equity is priced as a contingent claim on the bank asset value, the joint dynamics of the stock price and capital ratio is less tractable under the structural framework. It is also not straightforward to incorporate a jump in the stock price, which further restricts its ability to reflect the potential write-down of the CoCo bond value upon a conversion. On the other hand, the reduced form approach is efficient for pricing CoCos since it only requires the specification of the conversion intensity and the jump magnitude of the stock price at the conversion time. In reality, the capital ratio may be subject to manipulation by a distressed bank. The reduced form approach remains feasible to capture the possibility of regulatory trigger even when the capital ratio is not quite close to the triggering threshold. As such, the conversion can be treated as a rare event which can be approximated by a single Poisson jump. In this model framework, the CoCo bond is seen to be consisted of a straight bond component while the equity component is only a residual. A criticism of the reduced form approach is that it completely ignores the contractual feature of an accounting trigger and stays silent on the interaction between stock price and capital ratio.

In this paper, we propose a bivariate equity-credit modeling of the stock price and the capital ratio together with a jump-to-non-viability (JtNV) feature. This framework of a joint modeling is particularly important to pricing of CoCos since it only requires the specification of the conversion intensity and the jump magnitude of the stock price at the conversion time. In reality, the capital ratio may be subject to manipulation by a distressed bank. The reduced form approach remains feasible to capture the possibility of regulatory trigger even when the capital ratio is not quite close to the triggering threshold. As such, the conversion can be treated as a rare event which can be approximated by a single Poisson jump. In this model framework, the CoCo bond is seen to be consisted of a straight bond component while the equity component is only a residual. A criticism of the reduced form approach is that it completely ignores the contractual feature of an accounting trigger and stays silent on the interaction between stock price and capital ratio.
Schoutens (2012) can be recast under our bivariate framework when the stock price and capital ratio are perfectly correlated. This provides the justification for the equity derivative approach that replicates the CoCo components using barrier options. In our model, we add the reduced form feature that is important to capture a JtNV that is related to a sudden insolvency of the bank leading to a trigger. In the aftermath of the Lehman Brothers financial crisis, it is not uncommon to see substantial write downs by major investment banks due to unexpected trading losses and bleaching of regulation, which might erode a significant part of the bank’s capital. When the intensity is allowed to be state dependent and link to other economic variables, the JtNV can be used to model the sudden deterioration of a bank’s financial health leading to a PONV trigger. In summary, our proposed approach of integrating the reduced form approach and structural approach is natural for the pricing of a CoCo bond that has multiple sources of risk.

We note that the proposed framework is an extension of the equity-credit hybrid modeling framework that models the interaction between equity risk and credit risk (Carr and Linetsky, 2006; Carr and Wu, 2009; Cheridito and Wugalter, 2012; Chung and Kwok, 2014). For the sake of practical implementation, we adopt a model that is easy to implement and yet flexible enough to perform model calibration using the market prices of CoCos. We postulate a simple mean-reverting capital ratio process and model the accounting trigger by the first passage time of the capital ratio bleaching a predetermined threshold. Furthermore, we assume the stock price process to include the jump feature upon equity conversion process in order to capture the write-down in the stock price upon a sudden insolvency. To maintain analytical tractability, the first jump time of the stock price process is taken to be the JtNV trigger. Before the jump, the capital ratio and the stock price follow a bivariate Gaussian process. When the conversion intensity of the JtNV trigger is constant, we use an integral equation approach known as the Fortet method to compute the relevant density function of the first passage time to the triggering threshold of the capital ratio. When the conversion intensity is state dependent, the CoCo bond price can be obtained by solving a two-dimensional partial differential equation (PDE) numerically using the finite difference method. Though it is possible to extend to more sophisticated dynamics of stock price and capital ratio, like the inclusion of stochastic volatility or stochastic intensity, the infrequent observation and the lack of historical data of the capital ratio may not well justify the pursue of these advanced models.

This paper is structured as follows. Section 2 presents the model setup of the enhanced hybrid modeling approach. We present various structural features of the CoCo bond and explain why the joint distribution of the random time of equity conversion and the stock price at conversion time are crucial in the pricing procedure. In particular, we discuss the characterization of the jump in stock price upon equity conversion. The JtNV feature is modeled by the intensity of equity conversion. Both cases of constant intensity and state dependent intensity are considered. We illustrate the versatility of our pricing framework by showing how it can be used to cope with more complex structural features, like the floored payoff, coupon cancellation and perpetuity with callable feature. Section 3 presents the numerical examples that perform risk sensitivities analysis of a CoCo bond. We analyze the impact of various model parameters, like correlation, stock price volatility and conversion intensity on the price functions of the CoCo bonds with various contractual features. We also examine the conditional distribution of the stock price at an accounting trigger and analyze the decomposition of the CoCo bonds into their bond and equity components. Section 4 presents summary of results and conclusive remarks.
2 An Enhanced Hybrid Modeling

2.1 The structure

A typical structure of a CoCo bond consists of:

1. Bond component: coupon payments \((c_i)_{i=1,2,...,n}\) paid at time points \(t_i, i = 1,2,...,n\), and principal payment \(F\) at the maturity \(T\), where \(t_n = T\).

2. Equity component: at a trigger event, the bond is converted into \(G\) shares of equity of the issuing bank.

The equity exposure to the CoCo bond is revealed through equity conversion. In general, pricing of a CoCo bond is related to interest rate risk (discounting on the coupons and principal), equity risk (equity conversion at a trigger event) and conversion risk (loss absorption mechanism). The conversion risk is the risk of an unfavorable conversion to a declined stock price that wipes off a significant portion of the value of the bond. We emphasize that pricing of CoCos is not directly related to default risk since equity conversion always happens before a bank’s default. For pricing of CoCos, it suffices to have the knowledge of the stock price right after the conversion time.

Let the random conversion time be denoted by \(\tau\) and the stock price process by \(S = (S_t)_{t\geq 0}\). We assume a constant interest rate \(r\) and existence of a risk neutral measure \(Q\). The no-arbitrage price of a CoCo can be decomposed into three components:

\[
P_{CoCo} = P_C + P_F + P_E.
\]

1. Coupon payments \(P_C\):

\[
P_C = \sum_{i=1}^{n} E^Q [c_i e^{-rt_i} \mathbf{1}_{\{\tau > t_i\}}] = \sum_{i=1}^{n} c_i e^{-rt_i} [1 - Q(\tau \leq t_i)],
\]

which is the sum of discount coupon payments received prior to the conversion time \(\tau\).\(^1\)

2. Principal payment \(P_F\):

\[
P_F = E^Q [Fe^{-rT} \mathbf{1}_{\{\tau > T\}}] = Fe^{-rT} \{1 - Q(\tau \leq T)\},
\]

in which \(F\) is the principal payment when there is no conversion until maturity.

3. Conversion value \(P_E\):

\[
P_E = E^Q [e^{-r\tau} GS_{\tau} \mathbf{1}_{\{\tau \leq T\}}],
\]

where the CoCo is converted into \(G\) units of shares of the underlying equity at the conversion time \(\tau\), \(\tau = \tau_B \wedge \tau_R\).

It suffices to compute the conversion probability for evaluation of the bond component. On the other hand, the joint density of the stock price and conversion time is required to compute the conversion value \(P_E\). The key step in computing \(P_E\) is the determination of the joint modeling of the conversion time \(\tau\) and stock price \(S_t\). For conversion into cash, we only need to replace the term \(GS_{\tau}\) by a constant cash payment and this reduces an easier pricing problem. There are several other interesting contractual features, like the floored payoff, coupon cancellation and issuer’s call. All these features and their effects on the pricing procedures will be discussed in later sections.

\(^{1}\text{We ignore the small amount of interest accrual when a conversion occurs between two consecutive coupon payment dates.}\)


## 2.2 Model setup

We fix a filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, Q)\) in which \(Q\) is the risk neutral measure. We take the stock price process to be \(S_t = \exp(x_t)\) and capital ratio process to be \(H_t = \exp(y_t)\), where

\[X_t = (x_t, y_t)\] follows a bivariate process under the risk neutral measure \(Q\) as follows:

\[
\begin{align*}
    dx_t &= \left( r - q - \frac{\sigma^2}{2} \right) dt + \sigma dW^1_t + \gamma [dN_t - \lambda(x_t, y_t)dt], \quad x_0 = x, \\
    dy_t &= \kappa (\theta - y_t) dt + \eta \left( \rho dW^1_t + \sqrt{1 - \rho^2} dW^2_t \right), \quad y_0 = y.
\end{align*}
\]

(2.1)

Here, \(W^1_t\) and \(W^2_t\) are uncorrelated Brownian motions. The capital ratio is specified as an exponential Ornstein-Unlenbeck (OU) process with the mean reversion feature. This is because banks usually actively manage the amount of regulatory capital in response to the changing market values of asset and liability. They have the incentives to maintain a healthy level of capital ratio in order to stay away from any regulatory bleach. The interest rate \(r\), dividend yield \(q\), stock price volatility \(\sigma\), capital ratio volatility \(\eta\), correlation coefficient \(\rho\), mean reversion level \(\theta\) and speed of reversion \(\kappa\) of the capital ratio are assumed to be constant. The stock price process is modeled as a Geometric Brownian motion with jump upon equity conversion due to a sudden deterioration of the bank’s financial health or other distressed event leading to a PONV trigger. We let \(N_t\) denote the Poisson process that models the arrival of the PONV trigger and \(\lambda(x_t, y_t)\) be the state dependent intensity of \(N_t\). The constant jump magnitude of stock price upon PONV trigger is denoted by \(\gamma\).

Let \(H_B\) be the contractual threshold for an accounting trigger and define \(y_B = \ln H_B\). The accounting trigger is modeled by the first passage time of the log capital ratio \(y_t\) to a predetermined lower threshold \(y_B\) as

\[\tau_B = \inf \{ t \geq 0; y_t = y_B \} .\]

In addition, the random time of JtNV is modeled by the first jump of the Poisson process \(N_t\), where

\[\tau_R = \inf \{ t \geq 0; N_t = 1 \} .\]

The random time of equity conversion is taken to be the earlier of the first passage time \(\tau_B\) and the JtNV time \(\tau_R\), where

\[\tau = \tau_B \wedge \tau_R .\]

It is well acceptable to assume that \(\Pr \{ \tau_B = \tau_R \} = 0\) for the sake of simplicity. That is, the two random times do not occur at the same time almost surely.

The bivariate first passage time models with a one-sided threshold have been commonly adopted in modeling defaultable bonds and barrier options under stochastic interest rates (Longstaff and Schwartz, 1995; Collin-Dufresne and Goldstein, 2001; Coculescu et al., 2008; Bernard et al., 2008). Our proposed bivariate equity-credit models extend these bivariate first passage time models in several aspects. The accounting trigger is modeled by the first passage time of the mean reversion capital ratio hitting a threshold, which is typical in structural risky bond models. In addition, we adopt the reduced form approach of incorporating a random PONV trigger that is modeled by the first jump of a Poisson process and a jump in stock price upon PONV trigger. The novelty of choosing the first jump time \(\tau_R\) is that nice analytical tractability of the pricing model is maintained without pursuing a full partial-integral-differential equation (PIDE) formulation commonly seen in a jump-diffusion framework. When the intensity of the Poisson jump is constant, it is possible to simplify the pricing problem where one can employ an efficient numerical algorithm (Fortet method) to compute the CoCo bond price. When the intensity is chosen to be state dependent, the stock price and capital ratio processes interact through the correlation of the Brownian motions together.
with the state dependent jump intensity \( \lambda(x_t, y_t) \). This form of bivariate dependence of the jump intensity provides additional flexibility to model various triggering mechanisms in a CoCo bond. With some appropriate choices of the state dependent intensity functions, we manage to price the CoCo bonds by solving a two-dimensional PDE model using standard finite difference schemes.

It is worthwhile to discuss the jump in stock price upon an equity conversion. At the accounting trigger \( \tau = \tau_B \), the stock price is assumed to be continuous with no jump. The rationale is that the stock price should have gradually taken into account the possibility of such a conversion. At the regulatory trigger \( \tau = \tau_R \), there is a fixed jump in stock price as modeled by \( S_{\tau_R} = (1 + \gamma) S_{\tau_R} \), where \( \gamma \in (-\infty, 1) \). We rule out the case of \( \gamma = -1 \), which corresponds to jump to default. This is because the regulatory trigger is structured to avoid default of the issuing bank. Apparently, \( \gamma \leq 0 \) appears to be reasonable due to signaling of deterioration of bank’s financial health by the PONV trigger. The other case of \( \gamma > 0 \) may still be possible under the scenario of jump to recover.

In practice, it is not straightforward to determine the sign of the parameter \( \gamma \) since it is not sure whether the PONV trigger would enhance equity value or otherwise. The equity conversion helps reduce the liability of the bank by canceling the coupon and principal payments of the bond, which in turn boosts the capital adequacy and leads to a stronger balance sheet and stock price. On the other hand, the conversion into shares of stock causes dilution to existing equity holders which would be reflected by a weakened stock price. The actual effect of equity conversion on the stock price would become better known when there is an actual conversion event in the future. Lastly, our pricing model does not make any assumption on the change of the capital ratio after \( J_{tNV} \) as this information is irrelevant to pricing issues.

2.3 Conversion probability

As part of the pricing procedure, we show how to derive the conversion probability \( Q(\tau \leq t) \) under our bivariate equity-credit framework. It is necessary to consider the convolution of the two stopping times, \( \tau_B \) and \( \tau_R \). We make use of the following two building blocks:

\[
\begin{align*}
\mathbb{E}_Q[\mathcal{H}(\tau_B)1_{\{\tau_R > \tau_B\}}] &= \mathbb{E}_Q\left[e^{-\int_0^{\tau_B} \lambda_u du} \mathcal{H}(\tau_B)\right], \\
\mathbb{E}_Q[\mathcal{H}(\tau_R)1_{\{\tau_R < \tau_B\}}] &= \mathbb{E}_Q\left[\int_0^{\tau_B} \lambda_u e^{-\int_u^{\tau_B} \lambda_s ds} \mathcal{H}(u) du\right],
\end{align*}
\]

where \( \mathcal{H}(\xi) \) is the payoff at the corresponding stopping time \( \xi = \tau_R \) or \( \xi = \tau_B \). Note that the state dependent intensity \( \lambda_t = \lambda(x_t, y_t) \) introduces dependence among \( \tau_R \) and \( \tau_B \), which adds further complexity to the convolution.

**Lemma 1.** For a fixed \( t > 0 \), the probability of equity conversion is given by

\[
Q(\tau \leq t) = \int_0^t \mathbb{E}_Q\left[\lambda_u e^{-\int_0^u \lambda_s ds} 1_{\{\tau_B > u\}}\right] du + \mathbb{E}_Q\left[e^{-\int_0^{\tau_B} \lambda_u du} 1_{\{\tau_B \leq t\}}\right].
\]  \hspace{1cm} (2.2)

The proof of Lemma 1 is presented in Appendix A.1.

It is interesting to examine the conversion probability and the density function of the conversion time under our hybrid equity-credit framework. Firstly, the conversion density function is non-zero as \( t \to 0 \) due to the \( J_{tNV} \) feature (as revealed by the term \( \lambda e^{-\lambda t} \)) and this allows the possibility to capture the short-term credit spreads. Secondly, when the capital ratio is far away from the triggering threshold such that \( Q(\tau_B \leq t) \approx 0 \), we observe

\[
Q(\tau \leq t) \approx \mathbb{E}_Q\left[1_{\{\tau_R \leq t\}}\right] = Q(\tau_R \leq t),
\]
in which equity conversion arises solely from JtNV. In this case, we can treat the CoCo bond as a straight bond and one can incorporate a bond yield spread that is related to the jump intensity. This demonstrates the flexibility of the hybrid modeling framework to capture both the equity and fixed income nature of the CoCo bond.

In the next two subsections, we consider the pricing procedures of CoCo bonds under the assumption of constant intensity and state dependent intensity.

2.4 Constant intensity

Suppose the bond is converted into a predetermined $G$ units of shares at the conversion time, the conversion value is given by

$$P_E = G E^Q \left[ e^{-r \tau} S_\tau 1_{\{\tau \leq T\}} \right], \quad \tau = \tau_B \land \tau_R.$$

When the intensity $\lambda$ is constant, eq.(2.2) can be simplified as

$$Q(\tau \leq t) = \int_0^t \lambda e^{-\lambda u} \left[ 1 - Q(\tau_B \leq u) \right] du + E^Q \left[ e^{-\lambda \tau_B} 1_{\{\tau_B \leq t\}} \right].$$

The density function of the random conversion time can be expressed as

$$Q(\tau \in dt) = \lambda e^{-\lambda t} \left[ 1 - Q(\tau_B \leq t) \right] + e^{-\lambda t} Q(\tau_B \in dt).$$

Under constant intensity, the stopping times $\tau_B$ and $\tau_R$ are uncorrelated.

It is possible to apply a change-of-measure that simplifies the above pricing formula for $P_E$ by removing the dependence on the stock price $S$ (Cheridito and Xu, 2015). Under the stock price measure $Q^*$, the dimensionality of the pricing problem is reduced by one. We define the adjusted stock price process as follows:

$$\tilde{S}_t = S_t e^{-(r-q)t}, \quad = S_0 \exp \left( \int_0^t \sigma \, dW_1^s - \int_0^t \frac{\sigma^2}{2} \, ds - \gamma \lambda t \right) (1 + \gamma)^N_t, \quad t \geq 0.$$

It is readily to observe that the process $\tilde{S}_t$ is a positive martingale under the risk neutral measure $Q$. Hence, we can take $\tilde{S}_t$ to construct a change-of-measure as

$$Z_t = \frac{dQ^*}{dQ} \big|_{F_t} = \frac{\tilde{S}_t}{S_0},$$

where $Q^*$ can be interpreted as the stock price measure.

**Lemma 2.** Under the stock price measure $Q^*$, the bivariate process of $(x_t, y_t)$ evolves as

$$dx_t = \left( r - q - \frac{\sigma^2}{2} - \gamma \lambda \right) dt + \sigma \, dB^1_t + \gamma \, dN_t, \quad x_0 = x,$$

$$dy_t = \left[ \kappa (\theta - y_t) + \rho \sigma \eta \right] dt + \eta \, dB^2_t, \quad y_0 = y,$$

(2.3) where $\langle dB^1_t, dB^2_t \rangle = \rho \, dt$. The intensity of the Poisson process $N_t$ becomes

$$\lambda^* = (1 + \gamma) \lambda.$$
The proof of Lemma 2 is presented in Appendix A.2.

Now, we can apply the change-of-measure formula to compute the conversion value as follows:

\[
E^Q \left[ e^{-r\tau} S_\tau 1_{\{\tau \leq T\}} \right] = S_0 E^Q \left[ e^{-q \tau} e^{-q (r-q) \tau} S_\tau 1_{\{\tau \leq T\}} \right] = S_0 E^{Q^*} \left[ e^{-q \tau} 1_{\{\tau \leq T\}} \right],
\]

where \( \tau = \tau_B \wedge \tau_R \). As a result, we only need to compute a discounted conversion probability under the stock price measure \( Q^* \). The next two propositions provide the formulas to compute the conversion value.

**Proposition 3.** Denote \( \lambda^* = (1 + \gamma) \lambda \). The conversion value under constant intensity \( \lambda \) is given by

\[
P_E = G S_0 \left\{ \int_0^T \lambda^* e^{-(\lambda^*+q)u} [1 - Q^*(\tau_B \leq u)] \, du + E^{Q^*} \left[ e^{-\gamma (\lambda^*+q) \tau_B} 1_{\{\tau_B \leq T\}} \right] \right\}.
\]

**Proof.** By Lemma 2, it suffices to replace \( \lambda \) by \( \lambda^* = (1 + \gamma) \lambda \) and take the expectation under the stock price measure \( Q^* \). The remaining procedure is similar to that of Lemma 1.

By following a similar procedure of performing integration by parts (Campi et al., 2009) on the equity-credit hybrid modeling using a CEV process, we can obtain an alternative representation of the conversion value.

**Proposition 4.** When \( \lambda^* + q > 0 \), the conversion value can be expressed as

\[
P_E = G S_0 \left\{ \frac{\lambda^*}{\lambda^* + q} \left[ 1 - e^{-(\lambda^*+q)T} Q^*(\tau_B > T) \right] + \frac{q}{\lambda^* + q} E^{Q^*} \left[ e^{-(\lambda^*+q) \tau_B} 1_{\{\tau_B \leq T\}} \right] \right\}.
\]

**Proof.** Applying integration by parts and noting that \( \frac{\partial}{\partial t} Q^*(\tau_B \leq t) \) gives the density of \( \tau_B \), we obtain the result.

By Propositions 3 and 4, we have effectively reduced a two-dimensional problem into a one-dimensional problem using the change-of-measure formula. As a result, we only need to compute the distribution function of the first passage time \( Q^*(\tau_B \leq T) \) and the associated truncated Laplace transform

\[
E^{Q^*} \left[ e^{-\gamma (\lambda^*+q) \tau_B} 1_{\{\tau_B \leq T\}} \right].
\]

These two quantities can be readily computed by applying the one-dimensional Fortet method which can be summarized as follows. The details of the Fortet method are presented in Appendix B. We discretize the time interval \([0, T]\) into \( m \) equal intervals with \( t_j = j \Delta t \) for \( j = 1, 2, \ldots, m \), where \( m \Delta t = T \). The relevant quantities for the pricing of CoCo can be obtained by the following formulas:

\[
Q^*(\tau_B \leq t_m) = \sum_{j=1}^m q_j, \quad E^{Q^*} \left[ e^{-(\lambda^*+q) \tau_B} 1_{\{\tau_B \leq t_m\}} \right] = \sum_{j=1}^m e^{-(\lambda^*+q) t_j} q_j, \quad (2.4)
\]

where

\[
q_j = Q^*(\tau_B \in (t_{j-1}, t_j]], \quad j = 1, 2, \ldots, m,
\]

can be obtained by the following recursive scheme

\[
q_1 = N[a(t_1)],
\]

\[
q_j = N[a(t_j)] - \sum_{i=1}^{j-1} q_i N[b(t_j, t_i)], \quad j = 2, 3, \ldots, m,
\]

where \( a(t) \) and \( b(t, s) \) are defined in Appendix B.
identities:

\[ \rightpoint \]

right-point scheme as in Coculescu integration quadrature may lead to different forms of the recursive formula. Here, we present the 
an integral equation by discretization in the time domain, the different choices of the numerical integration quadrature may lead to different forms of the recursive formula. Here, we present the 
right-point scheme as in Coculescu et al. (2008), though one may choose alternative discretization schemes in order to achieve better rate of convergence.

2.5 State-dependent intensity

The nice analytical tractability in the direct computation of the conversion probability cannot be retained when we consider more general payoff structure upon equity conversion and/or the intensity becomes state dependent. We show how to formulate the pricing problem into a partial 
differential equation (PDE) formulation. Once the full characterization of the auxiliary conditions in the PDE formulation is known, one may compute the numerical solution of the pricing problem using standard finite difference method.

Since equity conversion occurs either at an accounting trigger or JtNV trigger, we may decompose the stock price at the conversion time \( \tau \) into the following form:

\[ S_\tau = S_{\tau B} 1_{\{\tau_R > \tau_B\}} + (1 + \gamma) S_{\tau R} 1_{\{\tau_B > \tau_R\}} \]

where there is a stock price jump at a JtNV trigger \( (\tau = \tau_R) \). By making use of the following identities:

\[ \mathbb{E}^Q \left[ e^{-\tau_B} G S_{\tau B} 1_{\{\tau_B \leq T\}} 1_{\{\tau_B < \tau_R\}} \right] = \mathbb{E}^Q \left[ e^{-\int_0^{\tau_B} (r + \lambda_u) \, du} G S_{\tau B} 1_{\{\tau_B \leq T\}} \right], \]

\[ \mathbb{E}^Q \left[ e^{-\tau_B} G (1 + \gamma) S_{\tau R} 1_{\{\tau_R \leq T\}} 1_{\{\tau_B < \tau_R\}} \right] = \mathbb{E}^Q \left[ \int_0^{\tau_B \wedge T} e^{-\int_0^u (r + \lambda_s) \, ds} \lambda_u (1 + \gamma) G S_u \, du \right], \]

where \( 1_{\{\tau_R \leq T\}} 1_{\{\tau_B < \tau_R\}} = 1_{\{\tau_R \leq \tau_B \wedge T\}} \) is used in the second identity, the conversion value can be expressed as

\[ P_E = \mathbb{E}^Q \left[ e^{-\int_0^{\tau_B} (r + \lambda_u) \, du} G S_{\tau B} 1_{\{\tau_B \leq T\}} + \int_0^{\tau_B \wedge T} e^{-\int_0^u (r + \lambda_s) \, ds} \lambda_u (1 + \gamma) G S_u \, du \right]. \quad (2.5) \]

Following a similar interpretation in the employee stock option model of Leung and Sircar (2009), the two terms in eq.(2.5) can be interpreted as

1. When \( y_t \) hits the barrier at \( \tau_B \), the investor receives the payoff \( G S_{\tau B} \);
2. When \( t < \tau_B \wedge T \), the investor receives the continuous cash-flow \( \lambda (1 + \gamma) G S_t \).

In our CoCo bond model, the barrier variable is the log capital ratio \( y_t \) while jump only happens in the log stock price process \( x_t \). Before the jump time, the joint process \( (x_t, y_t) \) is bivariate Gaussian. Therefore, we can compute the expectation by solving a PDE instead of a PIDE. A similar solution
technique has been used in solving an optimal stopping problem for jump-diffusion process with fixed jump size (Egami and Dayanik, 2012).

Next, we present the PDE formulation of the pricing function $P(x, y, t)$ of the conversion value of the CoCo bond. Given the joint dynamics of $(x_t, y_t)$ in eq.(2.1), the corresponding generator is given by

$$L = \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2} + \alpha \frac{\partial}{\partial x} + \rho \sigma \eta \frac{\partial^2}{\partial x \partial y} + \frac{\eta^2}{2} \frac{\partial^2}{\partial y^2} + \kappa (\theta - y) \frac{\partial}{\partial y}.$$  

Under certain mild technical conditions, the conversion value $P$ is the solution to the following Dirichlet problem

$$\frac{\partial P}{\partial t} + LP + \lambda(x, y)(1 + \gamma)Ge^x = [r + \lambda(x, y)]P,$$

for $(x, y, t) \in (-\infty, \infty) \times [y_B, \infty) \times [0, T]$. The boundary condition and terminal condition are

$$P(x, y_B, t) = Ge^x,$n
$$P(x, y_B, T) = \begin{cases} 
0, & y > y_B \\
Ge^x, & y = y_B 
\end{cases},$$

respectively. We assume the usual natural far field boundary conditions for $x \to \pm \infty$ and $y \to \infty$. The formulation in eq.(2.6) can be considered as an inhomogeneous PDE due to the non-linear term $\lambda(x, y)(1 + \gamma)Ge^x$, which represents the payoff due to the random termination at a JtNV trigger. On the other hand, the boundary condition at $y = y_B$ is indicated by the payoff at an accounting trigger.

We may model the different forms of dependence of the stock price and/or capital ratio on the financial state of the issuing bank through an appropriate choice of the state dependent intensity function. Some of these choices are discussed below.

1. Stock price dependent intensity:

$$\lambda(x) = \exp(a_0 - a_1 x), \quad a_1 > 0,$$

which prescribes an inverse relation between the intensity and stock price, with $\lambda(x, y) \to \infty$ as $S = e^x \to 0$. Das and Sundaram (2007) use a similar specification for a hybrid equity-credit modeling on convertible bonds. This specification takes the stock price as a measure of the financial health of the bank in which a low level of stock price indicates a high probability of PONV trigger. For implementation, it is possible to estimate the coefficients $a_0$ and $a_1$ by using the log-log plot of the stock price and the implied intensity of credit default swaps.

2. Capital ratio dependent intensity:

$$\lambda(y) = b_0 \mathbf{1}_{\{y \leq y_{RT}\}}, \quad b_0 > 0,$$

where $y_{RT}$ is a predetermined level that specifies the warning region in which the capital ratio is close to the contractual threshold such that the supervisory authority initiates the monitoring procedure of potential activation of the PONV trigger.

3. We can combine the above two specifications as a sum of two terms:

$$\lambda(x, y) = \exp(a_0 - a_1 x) + b_0 \mathbf{1}_{\{y \leq y_{RT}\}}, \quad a_1 > 0, \quad b_0 > 0.$$

This is seen to provide more flexible modeling of the PONV trigger.
2.6 Other contractual features

We would like to illustrate the versatibility of our pricing approach that it can accommodate more complex structural features of the CoCo bonds. We consider three examples of traded CoCo bond contracts with various payoff structures, like floored payoff, coupon cancellation and perpetuality with callable right.

2.6.1 Floored payoff

The issuer might impose a floor on the conversion value. For example, the Buffer Capital Note (BCN) issued by Credit Suisse has the conversion price limited at a floor of 20 USD per share. The floor feature is attractive from an investor’s perspective since this limits the downside risk of a CoCo bond that equity conversion occurs at a low stock price.

The conversion value with a floored payoff can be formulated as

$$P_{\text{Floor}} = G \mathbb{E}^Q \left[ e^{-r \tau} \max (S_\tau, K) 1_{\{\tau \leq T\}} \right], \quad \tau = \tau_B \wedge \tau_R,$$

where $K$ is the preset floor. As usual, by performing the following decomposition

$$\max (S_\tau, K) = \max (S_{\tau_B}, K) 1_{\{\tau_B > \tau_R\}} + \max ((1 + \gamma)S_{\tau_R}, K) 1_{\{\tau_R > \tau_B\}},$$

the conversion value with a floored payoff can be expressed as

$$P_{\text{Floor}} = G \mathbb{E}^Q \left[ e^{-\int_0^{\tau_B \wedge T} (r + \lambda_u) du} \max (S_{\tau_B}, K) 1_{\{\tau_B \leq T\}} ight. $$

$$\left. + \int_0^{\tau_B \wedge T} e^{-\int_0^u (r + \lambda_s) ds} \lambda_u \max ((1 + \gamma)S_u, K) du \right].$$

The corresponding PDE formulation is seen to be

$$\frac{\partial P}{\partial t} + \mathcal{L}P + \lambda(x, y)(1 + \gamma)G_{\text{Max}}((1 + \gamma)e^x, F) = (r + \lambda(x, y))P,$$

for $(x, y, t) \in (-\infty, \infty) \times [y_B, \infty) \times [0, T]$. The auxiliary conditions are given by

$$P(x, y_B, t) = G_{\text{Max}}(e^x, K),$$

$$P(x, y_B, T) = \begin{cases} 0, & y > y_B \\ G_{\text{Max}}(e^x, K), & y = y_B. \end{cases}$$

**Constant intensity**

Under the assumption that the intensity $\lambda$ is constant and $(1 + \gamma)S_{\tau_B} < K$, we can evaluate the conversion value with floored payoff using the extended Fortet method (see Appendix B.2 for full details). The key step involves the determination of the numerical approximation to the joint density of $(x_{\tau_B}, \tau_B)$ defined as follow:

$$Q (x_{\tau_B} \in dx, \tau_B \in dt) = q (x, t) dx dt.$$

In the numerical procedure, we discretize the domain for $x$ and $t$ using rectangular grids with the right-point scheme as:

$$x_i = x_{bi} + i \Delta x, \quad i = 1, 2, \ldots, n,$$

$$t_j = j \Delta t, \quad j = 1, 2, \ldots, m.$$
The numerical approximation for the joint density can be computed by the following recursive scheme

\[ q(x_i, t_1) = \Delta x \Phi(x_i, t_1), \quad j = 1, \]

\[ q(x_i, t_j) = \Delta x \left[ \Phi(x_i, t_j) - \sum_{h=1}^{j-1} \sum_{k=1}^{n} q(x_k, t_h) \psi(x_i, t_j; x_k, t_h) \right], \quad j = 2, 3, ..., m, \]

in which the analytic closed form formulas for \( \Phi(\cdot) \) and \( \psi(\cdot) \) are available for the bivariate Gaussian process (see Appendix B.2).

Once the joint density values at successive time points are known, the conversion value with a floored payoff can be evaluated by performing numerical integration of the following integral formula:

\[
P_{Floor} = G \int_0^T \int_{-\infty}^{\infty} q(x, u) e^{-(r+\lambda)u} \max(e^x, K) \, dx \, du + GK \int_0^T \lambda e^{-(r+\lambda)u} Q(\tau_B > u) \, du.
\]

Note that the optionality in \( \max((1+\gamma)S_{\tau_B}^\tau, K) \) is avoided by assuming sufficiently strong downward jump in stock price upon PONV trigger such that \((1+\gamma)S_{\tau_B}^\tau < K\).

### 2.6.2 Coupon cancellation

It is straightforward to modify our equity-credit framework to deal with other interesting features that appear in some issued CoCo bonds. One of these features is the coupon cancellation mechanism that entitles the issuer to the discretionary right to cancel the coupon payments before the conversion event. This can be considered as a partial conversion in which the issuing bank restructures her balance sheet by reducing the liability on future cash outflows. Corcuera et al. (2014) discuss the coupon cancellation feature of the CoCo bond issued by the Spanish bank BBVA, in which a coupon payment can be cancelled upon the sole discretion of the bank or the supervisory authority.

We can model the coupon cancellation (CC) event in a reduced form manner by introducing an independent exponential random variable \( \tau_C \sim \text{Exp}(\lambda_C) \). The CoCo bond price can be expressed as

\[
P_{EC}^{CC} = \sum_{i=1}^{n} \mathbb{E}^Q \left[ c_i e^{-rt_i} \mathbb{1}_{\{\tau_C > t_i\}} \right] + \mathbb{E}^Q \left[ F e^{-rT} \mathbb{1}_{\{\tau > T\}} \right] + \mathbb{E}^Q \left[ e^{-r\tau} G S_{\tau} \mathbb{1}_{\{\tau \leq T\}} \right], \quad \tau = \tau_B \wedge \tau_R,
\]

such that the coupon payments are related to the stopping time \( \tau_C \) only, while the principal payment and conversion value remain the same. The exogenous feature of \( \tau_C \) is also consistent with the discretionary nature of coupon deferral or coupon cancellation in reality. Assuming that the random time \( \tau_C \) is independent of the other random times, it is straightforward to extend to time-dependent intensity like those with piecewise constant intensity so that the term structure of coupon cancellation probability can be modeled.

### 2.6.3 Perpetual CoCo bond with callable feature

One innovative design of CoCos involves perpetual maturity along with a callable feature. For example, the Perpetual Subordinated Contingent Convertible Securities issued by the HSBC bank can be redeemed at par by the issuer on any coupon reset date. The major challenge in pricing these CoCo bonds lies in the modeling of the issuer’s call policy. As noted in Jarrow et al. (2010), one may approximate the issuing bank’s call policy using a reduced form approach from market
perspective. The reduced form approach of modeling call policy can be found in other applications, like mortgage prepayment and exercise of employee’s stock options.

Suppose that the CoCo bond is perpetual with a constant coupon stream \( c \) and a strike \( F \). We introduce an independent exponential random variable \( \tau_C \sim \text{Exp}(\lambda_C) \) as the random time of issuer’s call. This is seen to be equivalent to our equity-credit framework by setting \( \gamma = 0 \) and \( \lambda = \lambda_C \). In view of this observation, we can formulate the CoCo bond price as

\[
P_E^{\text{Callable}} = \sum_{i=1}^{\infty} \mathbb{E}^Q \left[ c e^{-rt_i} \mathbf{1}_{\{\tau_C \land \tau_B > t_i\}} \right] + \mathbb{E}^Q \left[ e^{-r\tau_C} GK \mathbf{1}_{\{\tau_B > \tau_C\}} \right] + \mathbb{E}^Q \left[ e^{-r\tau_B} GS_{\tau_B} \mathbf{1}_{\{\tau_C > \tau_B\}} \right],
\]

where \( K \) is the call price as specified in the contract. The first term is the coupon payment stream up to issuer’s call or accounting trigger, the second term is the discounted expected value when the issuer call before conversion while the third term is the discounted expected value when the accounting trigger occurs prior to a call. It is a common practice to assume that the issuer is going to exercise at the first call date, indicating that one can approximate the bond maturity by the first call date. Hence, we can set the intensity parameter \( \lambda_C \) such that the mean arrival time \( 1/\lambda \) is equal to the first call date.

3 Numerical Examples

3.1 Model parameters

For practical implementation of our bivariate equity-credit model, let us discuss about how to calibrate various model parameters from market prices of traded instruments. It may be plausible that the model price is insensitive to some of these model parameters, so we may preset the parameter values to assume some economically reasonable values instead of estimating them from market data. Given the contractual specification of a CoCo bond \((c, F, G)\), we estimate the model parameters \((\sigma, \rho, \eta, \kappa, \theta)\) for the bivariate process and \((\lambda, \gamma)\) for the embedded JtNV feature. The estimation procedures of the model parameters are summarized below.

- The stock price volatility \( \sigma \) can be estimated from historical time-series data or using the implied volatility of traded equity option.

- Since the capital ratio is observed on a regular basis, we can set the capital ratio \( y_0 \) accordingly and estimate the correlation coefficient \( \rho \) from historical data.

- The long-term mean level \( \theta \) and mean reversion speed \( \kappa \) can be estimated from the historical data of the capital ratio. Gupta et al. (2013) mention that major banks disclose the short-term and long-term target of the tier-1 capital ratio. One may calibrate a mean reversion model based on such information.

- We can calibrate the model to the market price of CoCo bond and estimate the market implied capital ratio volatility \( \eta \). In particular, the implied capital ratio volatility can be used to quantify how much conversion risk that the traders have priced in.

- The jump intensity \( \lambda \) can be related to other instruments such as the credit default swap and deep out-of-the-money put option. Since the CoCo conversion should occur prior to default of the issuing bank, the implied intensity of the credit default swap could be considered as a lower bound of the CoCo conversion intensity.
The determination of the jump size $\gamma$ is definitely not straightforward. We have discussed in Section 2.2 that both scenarios of positive and negative $\gamma$ are possible. Actual PONV trigger has yet to occur to indicate which scenario has a higher chance of occurrence.

In our sample calculations, we chose the following set of baseline parameters:

<table>
<thead>
<tr>
<th>$r$</th>
<th>$q$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$\eta$</th>
<th>$\kappa$</th>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.01</td>
<td>0.40</td>
<td>0.50</td>
<td>0.30</td>
<td>0.20</td>
<td>ln(0.10)</td>
<td>0.05</td>
<td>-0.70</td>
</tr>
</tbody>
</table>

We set a moderate and positive correlation coefficient to be 0.5 and take the stock price volatility to be 40%. For the capital ratio dynamics, we assume a long-term level of 10% with a mean reversion speed of 0.2, indicating that the bank takes around 5 years to adjust its capital ratio to its long-term mean level. We assume that the JtNV intensity is $\lambda = 0.05$ with the jump size $\gamma = -0.70$, indicating a high level of 70% write-down in stock price upon a PONV conversion. The intensity of 5% can be translated to a credit spread of roughly 350 bps using the rule-of-thumb: $s_{CDS} = \lambda (1 - R)$ with the recovery rate $R$ of 30%. We consider a CoCo bond with the following contractual specification: 5-year maturity, 10% coupon rate with the face value $F$ of $100, and the number of conversion shares $G$ is set to be 200 which implies the conversion price to be $0.5$. The accounting triggering level is set to be 5%, which gives $y_B = \ln 0.05$. Unless otherwise stated, the current stock price is set to be $S_0 = 0.5$.

We chose $a_1 = 0, 0.5$ or 1.0 in the functional form of the stock price dependent intensity and $a_0$ is set by matching $\lambda(x_0) = 0.05$. This allows us to have a benchmark comparison with the constant intensity case. Figure 1 illustrates the inverse relationship between the conversion intensity and stock price with varying values of $a_1$.

### 3.2 Check for numerical accuracy among various numerical schemes

In our sample calculations for checking numerical accuracy among various numerical schemes for pricing the CoCo bonds, we used the parameter values: $S_0 = 100$ and $G = 1$ for convenience. We set the time horizon to be $t = 1, 2, 3, 4, 5$. We calculated the conversion value $P_E$ using the Fortet method, explicit finite difference (FD) scheme and Monte Carlo simulation. For the FD scheme, we adopted 500 grid points in both the $x$- and $y$-direction and chose the natural boundary points to be sufficiently far such that the numerical results converge. In order to ensure that the explicit FD scheme is stable, we chose a small time step $\Delta t = 0.0001$. For the Monte Carlo simulation, we also took a small time step $\Delta t = 0.0001$ in order to capture the barrier hitting event effectively. The number of simulation paths was taken to be 100,000 in order to achieve sufficiently good accuracy. We implemented the Fortet method using the right point scheme and chose $\Delta t = 0.0001$ for consistency with other numerical schemes. Comparison of the numerical results using these three numerical methods are presented in Tables 1a, 1b and 1c. Good agreement of numerical results using various numerical schemes is confirmed.

### 3.3 Sensitivity analysis: constant intensity

We performed sensitivity analysis to examine how various model parameters may affect the CoCo bond price. First, we focus on the simple case with constant intensity and conversion into shares with no other complex contractual provision. In this case, the pricing formula can be expressed succinctly as

$$P = \sum_{i=1}^{n} c_i e^{-r t_i} \left[ 1 - Q(\tau \leq t_i) \right] + Fe^{-r T} \left[ 1 - Q(\tau \leq T) \right] + GS_0 E_{Q^*} \left[ e^{-\eta \tau} 1_{\{\tau \leq T\}} \right].$$

(3.1)
Here, $Q$ is the risk neutral measure and $Q^*$ is the stock price measure. The correlation coefficient $\rho$ and stock price volatility $\sigma$ enter into the pricing formula implicitly through the adjustment term added to the capital ratio dynamics under the stock price measure $Q^*$ in the conversion value $P_E$ term [the last term in eq.(3.1)]. In other words, the correlation coefficient and stock price volatility have no direct impact on the bond components $P_C$ and $P_F$ [the first two terms in eq.(3.1)].\footnote{This differs from the equity derivative approach in which the conversion time is proxied by the first passage time of the stock price to an implied threshold, in which the stock price volatility affects directly the CoCo bond price through the conversion probability.} From Lemma 2, we observe that the conversion value $P_E$ is related to the conversion probability under the stock price measure $Q^*$. When the correlation coefficient is positive, $\rho > 0$, the capital ratio dynamics has a higher mean reversion level due to the adjustment term $\rho \sigma \eta > 0$. This implies a smaller conversion probability under $Q^*$ and hence a smaller conversion value $P_E$. These theoretical observations are essential to understand the numerical plots of the pricing properties of the CoCo bonds presented below.

**Impact of correlation coefficient, $\rho$**

The joint density of the conversion time and stock price at an accounting trigger is highly dependent on the correlation coefficient $\rho$. When the correlation coefficient $\rho$ is more positive, the capital ratio exhibits stronger correlated movement with the stock price. As the capital ratio declines in value and hits the accounting trigger threshold, the stock price tends to stay at a lower level, thus resulting a smaller conversion value $P_E$. This explains why the CoCo price is a decreasing function of $\rho$. By comparing Figures 2a and 2b, we observe that the impact of $\rho$ on the CoCo price is more significant when the current capital ratio is further from the trigger threshold and the stock price volatility is higher. In Section 3.6, we present more detailed discussion of $\rho$ on the conditional distribution of the stock price at an accounting trigger and the resulting CoCo price.

**Impact of stock price volatility, $\sigma$**

Figure 3 shows that the CoCo bond price is decreasing with stock price volatility $\sigma$ when the correlation coefficient $\rho$ is positive. This pricing property is somewhat similar to that of a reverse convertible. When $\rho$ is zero, the CoCo bond price is insensitive to the stock price volatility (vega value is zero). This is expected since the conversion value is not affected by $\sigma$ due to the martingale property of the discounted stock price and the bond component is not affected by the stock price when the intensity is constant. The sensitivity of the CoCo price with respect to stock price volatility is stronger when the correlation coefficient is closer to one. Similar to a reverse convertible, there is no upside gain on the CoCo price when the stock price increases. However, the downside move of the stock price correlated with downside move in the capital ratio at positive correlation leads to a higher probability of activation of the loss absorption feature in the CoCo bond. This negative effect on the CoCo bond becomes more prominent when the stock price volatility is higher.

**Impact of conversion intensity, $\lambda$**

Figure 4 reveals the sensitivity of the CoCo bond price to the level of JtNV intensity $\lambda$ at different sizes of negative stock price jump $\gamma$ at conversion. We vary the jump size as $\gamma = -0.3, -0.5, -0.7$, which means that the write-down due to a JtNV is assumed to be 30%, 50% and 70%, respectively. Since the capital ratio of 8% is quite far away from the triggering threshold, most of the conversion risk of the CoCo bond lies at the possibility of a JtNV with the associated write down of the stock price and conversion value. The CoCo bond price can be highly sensitive to the intensity under the
assumption of a large jump size. In other words, the added JtNV feature provides the flexibility to incorporate a risk premium that lowers the value of a CoCo bond. This may explain the occurrence of low market price of the CoCo bond when the conversion probability generated by the capital ratio diffusion is relatively small.

3.4 Floored payoff

We would like to examine how the floored payoff may affect the conversion risk. In Figure 5, we show the CoCo bond price with different levels of stock price (delta risk) and stock price volatility (vega risk) with $K = 0.2$ and $K = 0.4$, corresponding to setting the floor to be 40% and 80% of the current stock price level $S_0 = 0.5$, respectively. For a high floor at $K = 0.4$, the CoCo bond price approaches the bond floor as the stock price falls close to the floor level of $K = 0.4$. For the vega risk, Figure 5(b) shows that the CoCo bond with a floored payoff is less sensitive to the stock price volatility and hence exhibits lower vega sensitivity. When we have the floor level set at a lower level $K = 0.2$, the floored payoff serves to protect the investor only at times with extremely low stock price. When the stock price level and volatility are at normal levels, the CoCo bond behaves like a standard CoCo bond without the floor and exhibits similar vega sensitivity. To summarize, the floored payoff provides protection for the investor from conversion risk and set a lower bound on the CoCo bond price. This generates a positive convexity of the CoCo bond price versus the stock price as well as a reduction on the vega sensitivity.

3.5 State dependent intensity

We explore the impact of stock price dependent intensity as exhibited by the following function

$$\lambda(x) = \exp(a_0 - a_1 x), \quad a_1 > 0;$$

in particular, the impact of the coefficient $a_1$ on the delta risk (see Figure 6). When $a_1 = 0$, the intensity function is reduced to the constant case and the CoCo bond price has a linear dependence to the stock price (due to the martingale property). When we introduce the stock price dependent intensity with $a_1 = 0.5$ and $a_1 = 1.0$, a negative convexity effect appears when the stock price is low. This is due to a higher probability of a JtNV trigger at low stock price such that the CoCo bond price becomes lower. On the other hand, the JtNV intensity decreases gradually as the stock price increases due to the inverse relationship. The CoCo bond prices converge to those under constant intensity ($a_1 = 0$) at high stock price. The incorporation of an inverse relation of the state dependent intensity on the stock price is important since the pricing model can generate negative convexity that models the death spiral effect in which equity sensitivity of a CoCo bond increases as the stock price is under distressed.

We would like to examine how the stock price dependent intensity induces additional vega sensitivity to the CoCo bond price. Figure 7(a) shows that when the correlation coefficient is zero, the CoCo bond price decreases with increasing stock price volatility. Recall from Figure 3 that the vega sensitivity is zero under constant intensity when $\rho = 0$. When the intensity has an inverse stock price dependence, a higher stock price volatility implies a higher chance of JtNV trigger since the stock price is more likely to diffuse to a lower level. As expected, the vega sensitivity is higher when we set a larger value of $a_1$. In Figure 7(b), we observe in a similar manner at $\rho = 0.5$ that stronger stock price dependence on the intensity leads to higher vega sensitivity of the CoCo bond price, though the impact is less when compared with the case where the correlation is zero.
3.6 Conditional distribution of the stock price at an accounting trigger

We would like to examine the conditional distribution of the stock price at an accounting trigger. This can be seen as an extension to the notion of implied trigger in Spiegeleer and Schoutens (2012) when there is an imperfect correlation between the stock price and capital ratio.

By the law of conditional probability, we have

\[ \Pr(x_{\tau_B} \in dx, \tau_B \in dt) = \Pr(x_{\tau_B} \in dx | \tau_B \in dt) \Pr(\tau_B \in dt), \]

so that the conditional distribution of \( x_t \) at the accounting trigger \( \tau_B \) is given by

\[ \Pr(x_{\tau_B} \in dx | \tau_B \in dt) = \frac{\Pr(x_{\tau_B} \in dx, \tau_B \in dt)}{\Pr(\tau_B \in dt)}. \] (3.2)

In order to examine the impact of varying levels of interaction between the stock price and capital ratio, we chose \( \rho = 0.3 \) and \( \rho = 0.9 \) in our sample calculations. The dispersion can be significant when \( \rho = 0.3 \). In Table 2, we observe that the mean of the implied stock price distribution can be much higher than the initial stock price of 0.5, indicating the possibility of significant divergence between the capital ratio and stock price at an accounting trigger. When \( \rho \) is 0.9, the mean of the implied distribution concentrates around and below 0.5. Figures 8(a) and 8(b) show that as the correlation coefficient increases, the implied stock price distribution becomes more concentrated. This indicates that under high value of \( \rho \), it becomes reasonable to infer the chance of an accounting trigger by monitoring the stock price level. This is consistent with the equity derivative approach that models the accounting trigger by the hitting time of the stock price to a lower barrier (down-and-out).

3.7 Decomposition into the bond and equity components

We examine the decomposition of the CoCo bond value into its bond and equity components with varying values of the capital ratio. This serves to illustrate the change in equity exposure of the CoCo bond as the capital ratio increases.

As revealed in Figure 9, the proportion of the equity component can be almost half of the CoCo bond value when the capital ratio is close to the accounting trigger threshold of 5%. This highlights the importance to model the interaction of stock price and capital ratio by the bivariate dynamics. When there is no JtNV (\( \lambda = 0 \)), the equity part of a CoCo bond becomes small as the capital ratio increases to 12%, indicating diminishing equity exposure. When there exists more significant JtNV risk (\( \lambda = 0.05 \)), we observe a residual equity component of the CoCo bond even when the capital ratio increases to 12%. This is consistent with the market consensus that the CoCo bond remains to have some equity exposure.

4 Conclusion

We propose a hybrid equity-credit model with underlying bivariate process of stock price and capital ratio, together with equity conversion feature that is subject to accounting trigger and point-of-non-viability (PONV) trigger. The accounting trigger is modeled using the structural approach as the event of the capital ratio hitting a preset contractual threshold value from above. On the other hand, we model the PONV trigger by the reduced form approach and consider it as a Poisson arrival with specified conversion intensity. We consider various specifications of the contractual payoff upon equity conversion from actual CoCo bond contracts. We also discuss different functional forms of state dependence of the conversion intensity on the stock price and capital ratio. For general payoff
structure upon equity conversion and state dependent conversion intensity under PONV trigger, we manage to formulate the pricing problem as a two-dimensional partial differential equation (PDE). The numerical solution of the governing PDE can be computed using standard finite difference scheme. Under certain simplifying assumption on the conversion payoff and constant conversion intensity, we reduce the pricing problem to evaluation of the conversion probability. We show how to construct efficient numerical algorithms using the Fortet method to compute prices of the CoCo bonds. We also discuss various issues in the calibration of the model parameters from market prices of traded instruments. The pricing properties of the CoCo bonds under various market conditions and the sensitivity analysis of the price functions under varying values of the model parameters are examined. Our bivariate equity-credit model provides a flexible pricing framework that can incorporate various modeling features and contractual specifications of CoCo bonds traded in the market.

References


A Proof of Lemmas

A.1 Lemma 1

We consider a general stochastic intensity \( \lambda_t = \lambda(X_t) \) with Markov process \( X_t \in \mathbb{R}^2 \). Since \( \tau = \tau_B \wedge \tau_R \), the decomposition into the two events \( \{ \tau_B > \tau_R \} \) and \( \{ \tau_R > \tau_B \} \) gives

\[
Q(\tau \leq t) = \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_B \wedge \tau_R \leq t\}} \right] = \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_B \leq t\}} \mathbb{1}_{\{\tau_B > \tau_R\}} \right] + \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_B \leq t\}} \mathbb{1}_{\{\tau_R > \tau_B\}} \right].
\]

The first term corresponds to the scenario where the JtNV trigger occurs prior to the accounting trigger. By virtue of iterated expectation, we obtain

\[
\mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_R \leq t\}} \mathbb{1}_{\{\tau_B > \tau_R\}} \right] = \mathbb{E}^Q \left[ \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_R \leq t\}} \mathbb{1}_{\{\tau_B > \tau_R\}} \mid \tau_R = u \right] \right] = \int_0^t \mathbb{E}^Q \left[ \lambda(X_u) e^{-\int_0^u \lambda(X_s) \, ds} \mathbb{1}_{\{\tau_B > \tau_R\}} \right] \, du.
\]

The second term gives the conversion probability conditional on accounting trigger occurring prior to the JtNV trigger. It is straightforward to obtain

\[
\mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_R \leq t\}} \mathbb{1}_{\{\tau_R > \tau_B\}} \right] = \mathbb{E}^Q \left[ e^{-\int_0^\tauB \lambda(X_u) \, du} \mathbb{1}_{\{\tau_B \leq t\}} \right].
\]

A.2 Lemma 2

The proof follows from Lemma 11.6.1 and Theorem 11.6.2 in Shreve (2004). From the adjusted stock price process \( \tilde{S}_t \), we can decompose the Radon-Nikodym density as

\[
Z_t = Z^c_t Z^J_t,
\]

where

\[
Z^c_t = \exp \left( \int_0^t \sigma \, dW^1_s - \int_0^t \frac{\sigma^2}{2} \, ds \right) \quad \text{and} \quad Z^J_t = \exp \left( -\gamma \lambda t \right) (1 + \gamma)^{N_t},
\]

corresponding to the change-of-measure for the continuous path and jump path, respectively. Since the Brownian motion is only affected by the change-of-measure \( Z^c_t \), by the Girsanov theorem, we have

\[
 dB^1_t = dW^1_t - \sigma \, dt.
\]

The dynamic equation for \( y_t \) under the stock price measure \( Q^* \) is given by

\[
d y_t = \kappa (\theta - y_t) \, dt + \eta \left( \sqrt{1 - \rho^2} \, dW^2_t + \rho \, dW^1_t \right) = \left[ \kappa (\theta - y_t) + \rho \sigma \eta \right] \, dt + \eta \, dB^2_t,
\]

where \( dB^2_t = \sqrt{1 - \rho^2} dW^2_t + \rho \, dB^1_t \) and \( \langle dB^1_t, dB^2_t \rangle = \rho \, dt \). On the other hand, the Poisson process is affected only by the change-of-measure \( Z^J_t \) as

\[
Z^J_t = \exp \left( -\gamma \lambda t \right) (1 + \gamma)^{N_t} = \exp \left( (\lambda^* - \lambda^*) t \right) \frac{\lambda^*}{\lambda}^{N_t},
\]

where the second equality is derived from the change-of-measure for a Poisson process with constant intensity. Hence, the intensity becomes \( \lambda^* = (1 + \gamma) \lambda \) under the stock price measure \( Q^* \).
B Fortet Method

B.1 One-dimensional Fortet scheme

Consider the following dynamic equation of the capital ratio $y_t$ under the stock price measure $Q^*$, where

$$\text{d}y_t = \left[ \kappa (\theta - y_t) + \rho \sigma \eta \right] \text{d}t + \eta \, dB_t.$$ 

Let $\tau_B$ denote the first passage time of $y_t$ hitting the threshold $y_B$ and $q(t)$ be the corresponding density function defined by $\Pr \{ \tau_B \in dt \} = q(t) \, dt$. The transition probability density for the process $y_t$ conditional on $s < t$ is defined by

$$\Pr \{ y_t \in dy | y_s \in dy' \} = f(y,t; y',s) \, dy.$$ 

Suppose the capital ratio process starts at $y_0$ at $t = 0$ and moves downstream to $y$ at time $t$, where $y < y_B < y_0$, then the process must cross the threshold $y_B$ at some time prior to time $t$ as it moves from $y_0$ to $y$. By the continuity and strong Markov property of the process, we observe

$$f(y,t; y_0,0) = \int_0^t q(s) \, f(y,t; y_B,s) \, ds.$$ 

Integrating $y$ on both sides over $(-\infty, y_B]$, we obtain the Fortet equation as follows:

$$N \left[ \frac{y_B - \mu(t,0)}{\Sigma(t,0)} \right] = \int_0^t q(s) \, N \left[ \frac{y_B - \mu(t,s)}{\Sigma(t,s)} \right] \bigg|_{y_s=y_B} \, ds,$$ 

where

$$\mu(t,s) = y_s e^{-\kappa(t-s)} + \left( \theta + \frac{\rho \sigma \eta}{\kappa} \right) \left[ 1 - e^{-\kappa(t-s)} \right], \quad \Sigma^2(t,s) = \frac{\eta^2}{2\kappa} \left[ 1 - e^{-2\kappa(t-s)} \right].$$

We write

$$a(t) = \frac{y_B - \mu(t,0)}{\Sigma(t,0)}, \quad b(t,s) = \frac{y_B - \mu(t,s)}{\Sigma(t,s)} \bigg|_{y_s=y_B},$$

so that eq. (B.1) can be expressed as

$$N[a(t)] = \int_0^t q(s) \, N[b(t,s)] \, ds.$$ 

This is seen to be a Volterra integral equation of the first kind. To solve the integral equation numerically, we apply the right point scheme of numerical integration. By approximating the integral over the discrete time points, $t_j = j \Delta t$, $j = 1, \ldots, m$, where $\Delta t$ is the uniform time step, we obtain

$$N[a(t_j)] = \sum_{i=1}^j q_i \, N[b(t_j,t_i)].$$

Here, we have chosen the discrete approximation

$$q_j \approx \Pr \{ \tau_B \in (t_{j-1},t_j] \}, \quad j = 1, 2, \ldots, m.$$ 

We then deduce the following recursive scheme for calculating $q_j$, $j = 1, 2, \ldots, m$, successively (Longstaff and Schwartz, 1995; Coculescu et al., 2008):

$$q_1 = N[a(t_1)],$$

$$q_j = N[a(t_j)] - \sum_{i=1}^{j-1} q_i \, N[b(t_j,t_i)], \quad j = 2, 3, \ldots, m.$$
B.2 Two-dimensional case

The calculation for floored payoff and the conditional stock price distribution require the joint density of \((x_{\tau_B}, \tau_B)\) under the risk neutral measure \(Q\). The joint dynamics of \((x_t, y_t)\) conditional on \((x_s, y_s), s < t\), and the event \(\{\tau_R > t\}\) can be written as

\[
\begin{align*}
x_t &= x_s + \left(r - q - \frac{1}{2} \sigma^2 - \lambda \gamma\right) (t - s) + \int_s^t \sigma \, dB_u^1 , \\
y_t &= y_s e^{-\kappa(t-s)} + \theta \left[1 - e^{-\kappa(t-s)}\right] + \int_s^t \eta e^{-\kappa(t-u)} dB_u^2 ,
\end{align*}
\]

where \(\langle dB^1_t, dB^2_t \rangle = \rho \, dt\). Note that an adjustment term \(-\lambda \gamma\) is added in the drift term of \(x_t\) due to the constant intensity jump upon equity conversion. We would like to apply the Fortet method to find the numerical approximation of the joint density \(q(x, t)\) of \((x_{\tau_B}, \tau_B)\) as defined by

\[
\Pr \{x_{\tau_B} \in dx, \tau_B \in dt\} = q(x, t) \, dx \, dt.
\]

Let \(f(y, x, t; y', x', s)\) denote the transition probability density for the bivariate process \((y_t, x_t)\) conditional on \((x_s, y_s), s < t\), where

\[
\Pr \{y_t \in dy, x_t \in dx | y_s \in dy', x_s \in dx'\} = f(y, x, t; y', x', s) \, dy \, dx.
\]

Similar to the one-dimensional case, suppose \(y_t\) starts at \(y_0\) at \(t = 0\) and moves downstream to \(y\) at time \(t\), then the process crosses \(y_B\) at some time prior to time \(t\), where \(y < y_B < y_0\). By the continuity and strong Markov property of the joint process \((y_t, x_t)\), we have

\[
f(y, x, t; y_0, x_0, 0) = \int_0^t \int_{-\infty}^{\infty} q(x', s) f(y, x, t; y_B, x', s) \, dx' \, ds.
\]

Integrating \(y\) on both sides over \((-\infty, y_B]\), we obtain the extended Fortet equation as follows:

\[
\Phi(x, t) = \int_0^t \int_{-\infty}^{\infty} q(x', s) \psi(x, t; x', s) \, dx' \, ds,
\]

where the marginal distribution functions are given by

\[
\Phi(x, t) = \int_{-\infty}^{y_B} f(y, x, t; y_0, x_0, 0) \, dy, \quad \psi(x, t; x', s) = \int_{-\infty}^{y_B} f(y, x, t; y_B, x', s) \, dy,
\]

and whose closed form expressions will be presented later. Since \(y_0 > y_B\), it is seen that

\[
\lim_{t \to 0} \Phi(x, t) = 0, \quad \lim_{t \to s} \psi(x, t; x', s) = \delta(x - x'),
\]

where \(\delta(\cdot)\) is the Dirac delta function. In order to solve the two-dimensional integral equation (B.2) numerically, we discretize the domain for \(x\) and \(t\) using a set of rectangular grids as follows:

\[
x_i = x_{iB} + i \Delta x, \quad i = 1, 2, \ldots, n, \quad \text{and} \quad t_j = j \Delta t, \quad j = 1, 2, \ldots, m.
\]

We approximate the integral equation (B.2) by

\[
\Phi(x_i, t_j) = \sum_{\ell=1}^{j} \sum_{k=1}^{n} q(x_k, t_\ell) \psi(x_i, t_j; x_k, t_\ell),
\]
where \(q(x_i, t_j)\) denotes the discrete approximation

\[
q(x_i, t_j) \approx \Pr \{x_{\tau B} \in (x_{i-1}, x_i), \tau_B \in (t_{j-1}, t_j)\}.
\]

For \(j = 1\), we obtain the approximation formula

\[
\Delta x \Phi(x_i, t_1) = \Delta x \sum_{k=1}^{n} q(x_k, t_1) \psi(x_i, t_1; x_k, t_1) = q(x_i, t_1),
\]

using the properties in eq.(B.3). For \(j > 1\), we obtain the approximation scheme

\[
\Delta x \Phi(x_i, t_j) = \Delta x \sum_{k=1}^{n} q(x_k, t_j) \psi(x_i, t_j; x_k, t_j) + \Delta x \sum_{\ell=1}^{j-1} \sum_{k=1}^{n} q(x_k, t_\ell) \psi(x_i, t_j; x_k, t_\ell).
\]

By eq.(B.3), the first term on the right hand side gives \(q(x_i, t_j)\). As a result, we have the following recursive scheme for finding \(q(x_i, t_j)\) at successive time points \(t_j, j = 1, 2, \ldots, n\), and at varying level of \(x_i, i = 1, 2, \ldots, n:\)

\[
q(x_i, t_1) = \Delta x \Phi(x_i, t_1),
\]

\[
q(x_i, t_j) = \Delta x \left[ \Phi(x_i, t_j) - \sum_{\ell=1}^{j-1} \sum_{k=1}^{n} q(x_k, t_\ell) \psi(x_i, t_j; x_k, t_\ell) \right], \quad j = 2, 3, \ldots, m. \tag{B.4}
\]

Lastly, we derive the closed form expressions for \(\Phi\) and \(\psi\) in eq.(B.4). To this end, we observe that \(y_t|_{x_t, x_s} \sim N(\mu(t; s), \Sigma^2(t; s))\) is Gaussian and whose the conditional moments under the risk neutral measure \(Q\) can be obtained by the projection theorem as follows:

\[
\mu(t; s) \triangleq \mathbb{E}_s[y_t|x_t] = \mathbb{E}_s[y_t] + \frac{\text{cov}_s(y_t, x_t)}{\text{var}_s[x_t]} [x_t - \mathbb{E}_s[x_t]],
\]

\[
\Sigma^2(t; s) \triangleq \text{var}_s[y_t|x_t] = \text{var}_s[y_t] - \frac{\text{cov}_s(y_t, x_t)^2}{\text{var}_s[x_t]}.
\]

The unconditional moments are readily obtained as

\[
\mathbb{E}_s[x_t] = x_s + \left( r - q - \frac{1}{2} \sigma^2 - \lambda \gamma \right) (t - s), \quad \mathbb{E}_s[y_t] = y_se^{-\kappa(t-s)} + \theta \left[ 1 - e^{-\kappa(t-s)} \right],
\]

\[
\text{var}_s[x_t] = \sigma^2 (t - s), \quad \text{var}_s[y_t] = \frac{\eta^2}{2\kappa} \left[ 1 - e^{-2\kappa(t-s)} \right], \quad \text{cov}_s(y_t, x_t) = \rho \sigma \eta \left[ \frac{1 - e^{-\kappa(t-s)}}{\kappa} \right].
\]

The conditional probability relation gives

\[
f(y_t, x_t, t; y_s, x_s, s) = f(x_t, t; x_s, s) f(y_t, t; y_s, x_s, s|x_t),
\]

where

\[
f(x_t, t; x_s, s) = \frac{1}{\sqrt{2\pi \sigma^2(t-s)}} \exp \left( -\frac{[x_t - x_s - (r - q - \frac{1}{2} \sigma^2 - \lambda \gamma)(t-s)]^2}{2\sigma^2(t-s)} \right),
\]

\[
f(y_t, t; y_s, x_s, s|x_t) = \frac{1}{\sqrt{2\pi \Sigma^2(t; s)}} \exp \left( -\frac{[y_t - \mu(t; s)]^2}{2\sigma^2 \Sigma^2(t; s)} \right).
\]
The integration of the above conditional density formula with respect to \( y \) over \((-\infty, y_B)\) gives the following closed form expressions for \( \Phi(x,t) \) and \( \psi(x,t;x',s) \):

\[
\Phi(x,t) = f(x_t,t;x_0,0) N \left[ \frac{y_B - \mu(t;0)}{\Sigma(t;0)} \right],
\]

\[
\psi(x,t;x',s) = f(x_t,t;x_s,s) N \left[ \frac{y_B - \mu(t;s)}{\Sigma(t;s)} \right] \bigg|_{y_s=y_B}.
\]
Table 1a: Numerical calculations of the conversion value with constant intensity $\lambda = 0.05$ and varying values of the capital ratio using the Fortet method, finite difference (FD) scheme and Monte Carlo simulation method. The bracket quantities are the standard errors of the Monte Carlo simulation results. Good agreement of the numerical results using different numerical methods is observed.
(i) Capital ratio = 6%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.720</td>
<td>37.170 (0.138)</td>
</tr>
<tr>
<td>2</td>
<td>46.472</td>
<td>46.029 (0.141)</td>
</tr>
<tr>
<td>3</td>
<td>50.827</td>
<td>50.381 (0.141)</td>
</tr>
<tr>
<td>4</td>
<td>53.674</td>
<td>53.300 (0.141)</td>
</tr>
<tr>
<td>5</td>
<td>55.785</td>
<td>55.400 (0.142)</td>
</tr>
</tbody>
</table>

(ii) Capital ratio = 8%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.591</td>
<td>6.511 (0.063)</td>
</tr>
<tr>
<td>2</td>
<td>14.379</td>
<td>14.141 (0.090)</td>
</tr>
<tr>
<td>3</td>
<td>19.714</td>
<td>19.663 (0.103)</td>
</tr>
<tr>
<td>4</td>
<td>23.666</td>
<td>23.623 (0.112)</td>
</tr>
<tr>
<td>5</td>
<td>26.795</td>
<td>26.795 (0.118)</td>
</tr>
</tbody>
</table>

(iii) Capital ratio = 10%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.209</td>
<td>2.172 (0.030)</td>
</tr>
<tr>
<td>2</td>
<td>6.503</td>
<td>6.385 (0.056)</td>
</tr>
<tr>
<td>3</td>
<td>10.643</td>
<td>10.657 (0.073)</td>
</tr>
<tr>
<td>4</td>
<td>14.193</td>
<td>14.152 (0.085)</td>
</tr>
<tr>
<td>5</td>
<td>17.230</td>
<td>17.380 (0.094)</td>
</tr>
</tbody>
</table>

Table 1b: Numerical calculations of the conversion value with state dependent intensity specified as \( \lambda(x) = \exp(a_0 - a_1 x) \), where \( a_1 = 0.5 \) and \( a_0 \) is set by matching \( \lambda(x_0) = 0.05 \). Good agreement of the numerical results using the finite difference (FD) scheme and Monte Carlo simulation method is observed. The bracket quantities are the standard errors of the Monte Carlo simulation results.
(i) Capital ratio = 6%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38.701</td>
<td>38.364 (0.139)</td>
</tr>
<tr>
<td>2</td>
<td>47.540</td>
<td>47.158 (0.143)</td>
</tr>
<tr>
<td>3</td>
<td>51.900</td>
<td>51.701 (0.143)</td>
</tr>
<tr>
<td>4</td>
<td>54.734</td>
<td>54.335 (0.144)</td>
</tr>
<tr>
<td>5</td>
<td>56.828</td>
<td>56.704 (0.146)</td>
</tr>
</tbody>
</table>

(ii) Capital ratio = 8%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.932</td>
<td>6.913 (0.064)</td>
</tr>
<tr>
<td>2</td>
<td>14.915</td>
<td>14.846 (0.092)</td>
</tr>
<tr>
<td>3</td>
<td>20.342</td>
<td>20.389 (0.106)</td>
</tr>
<tr>
<td>4</td>
<td>24.343</td>
<td>24.198 (0.113)</td>
</tr>
<tr>
<td>5</td>
<td>27.501</td>
<td>27.506 (0.120)</td>
</tr>
</tbody>
</table>

(iii) Capital ratio = 10%

<table>
<thead>
<tr>
<th>Time</th>
<th>FD Scheme</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.279</td>
<td>2.293 (0.031)</td>
</tr>
<tr>
<td>2</td>
<td>6.703</td>
<td>6.643 (0.056)</td>
</tr>
<tr>
<td>3</td>
<td>10.938</td>
<td>10.873 (0.073)</td>
</tr>
<tr>
<td>4</td>
<td>14.554</td>
<td>14.722 (0.087)</td>
</tr>
<tr>
<td>5</td>
<td>17.639</td>
<td>17.584 (0.095)</td>
</tr>
</tbody>
</table>

Table 1c: Numerical calculations of the conversion value with state dependent intensity specified as $\lambda(x, y) = \exp(a_0 - a_1 x) + b_0 \mathbf{1}_{\{y \leq y_{RT}\}}$, where $a_1 = 0.5$, $a_0$ is set by matching $\lambda(x_0) = 0.05$, $b_0 = 0.1$ and $y_{RT} = 0.07$. Good agreement of the numerical results using the finite difference (FD) scheme and Monte Carlo simulation method is observed. The bracket quantities are the standard errors of the Monte Carlo simulation results.

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0.3$</td>
<td>0.48</td>
<td>0.58</td>
<td>0.69</td>
<td>0.82</td>
<td>0.96</td>
</tr>
<tr>
<td>$\rho = 0.9$</td>
<td>0.28</td>
<td>0.31</td>
<td>0.34</td>
<td>0.38</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 2: Conditional mean of the stock price distribution at an accounting trigger. At $\rho = 0.3$, the mean of the implied stock price distribution can be much larger than the initial stock price of 0.5.
Figure 1: Inverse relationship between conversion intensity and stock price with varying values of $a_1$ in the state dependent intensity function.

Figure 2: Impact of correlation coefficient $\rho$ on the CoCo bond price.

(a) Capital ratio = 5.125%.

(b) Capital ratio = 8%.

Figure 3: Impact of stock price volatility $\sigma$ on the CoCo bond price with capital ratio of 8%.
Figure 4: Impact of JtNV intensity $\lambda$ on the CoCo bond price with capital ratio of 8%.

Figure 5: Impact of floored payoff on the CoCo bond price.

Figure 6: Effect of stock price dependent intensity on delta risk.
Figure 7: Effect of stock price dependent intensity on vega risk.

(a) Zero correlation.

(b) Correlation coefficient = 0.5.

Figure 8: Plots of the conditional distribution of stock price at an accounting trigger with varying values of the correlation coefficient.

(a) Correlation coefficient, $\rho = 0.3$.

(b) Correlation coefficient, $\rho = 0.9$. 
(a) The JtNV intensity is set to zero: the equity exposure diminishes when the capital ratio stays around 12%.

(b) The JtNV intensity is set to be 5%: the equity exposure remains to be significant even when the capital ratio stays around 12%.

Figure 9: Decomposition of the CoCo bond value into its bond and equity components.