Preface

Objectives and Audience

Most problems in pricing financial derivatives and risk measures calculation in credit portfolios involve computation of tail probabilities and tail expectations of the corresponding underlying state variables. Financial institutions always strive for effective valuation of prices of exotic financial derivatives and risk positions of portfolios of risky instruments. It is desirable to derive efficient, reliable and accurate analytic approximation formulas for derivatives pricing and risk measures computation. The saddlepoint approximation approach has been shown to be a versatile tool in statistics that is based on the steepest descent method to derive analytic approximation formulas of the Laplace inversion integrals for density functions, tail probabilities and tail expectation of a sum of independent and identically distributed random variables. There has been a growing literature in recent decades in applying the saddlepoint approximation methods in pricing exotic financial derivatives and calculating risk measures and risk contributions in credit portfolios under various default correlation models.

In this book, we summarize recent research results in applying the saddlepoint approximation methods in pricing exotic derivatives and calculating risk measures of credit portfolios. We start with the presentation of derivation of a variety of saddlepoint approximation formulas under different applications so that researchers new to the topic may comprehend the fine technicalities in various saddlepoint approximation approaches. The exposition and style are made rigorous by providing formal proofs of most of the results. Also, this book is made to be self-contained by providing the relevant background knowledge of the analytic tools used in the derivation procedures found in Appendices of the chapters. Illustrative numerical examples are included to help readers make assessment on accuracy and effectiveness of the saddlepoint approximation formulas under various applications. In a few cases, we show how some saddlepoint approximation formulas may fail to produce accurate results.

This book will be valuable to researchers in saddlepoint approximation since it offers a single source for most of the saddlepoint approximation results in financial engineering, with different sets of ready-to-use approximation formulas. Many of these results may otherwise only be found in the original research publications. It is also well suited as a text book on the subject for courses in financial mathematics at universities.

Guide to the Chapters

This book contains five chapters. The foundations are laid in the first chapter in terms of generalized Fourier transform (or characteristic function) of the probability density function of a random variable and the inverse Fourier transform, by which the probability density function can be recovered from the characteristic function. The Bromwich integral formulas of the density, tail probability and tail expectation are expressed in terms of the cumulant generating function of the underlying random variable. The steepest descent method is applied to obtain the asymptotic expansion of a Fourier or Laplace integral. As the basic essence of the saddlepoint approximation approach, a Bromwich contour in the complex plane is deformed to pass through a saddlepoint along the path of steepest descent.

In Chapter Two, we present the detailed procedures of deriving various saddlepoint approximation formulas for the density functions, tail probabilities and tail expectations of random variables under both the Gaussian and non-Gaussian base distribution, using the steepest descent method and the technique of exponentially titled Edgeworth expansion. In this chapter, the saddelpoint approximation formulas are derived based on the assumption that the cumulant generating function of the underlying random variable is known in closed from. Beyond the theoretical derivation and numerical examples, we explore the scenarios under which some saddlepoint approximation formulas may fail. Since an Edgeworth expansion is expanded around the mean, it is envisioned that expansion may fail in the tail of the distribution. For this reason, the exponentially tilted Edgeworth expansions are considered, in which one performs the expansion in the region of interest. This would lead to more accurate saddlepoint approximations for tail probabilities and tail expectations, which are of great use in credit risk portfolio models and option pricing in later chapters.

In Chapter Three, the saddlepoint methods from two research papers are presented for continuous time Markov processes of affine-jump diffusion, in which case a closed form solution for the cumulant generating function is not available. In the first section of the chapter, a Taylor expansion in small time is used to obtain the cumulant generating function and the saddlepoint. An error analysis of the derived approximation is provided. In the second section, the focus is on the affine jump-diffusion model in which the characteristic function of the process has an exponential affine structure. The characteristic function can be solved in terms of a system of Ricatti ordinary differential equations, either explicitly or numerically. On this basis, the saddlepoint can be approximated by combining a root finding algorithm, together with the series inversion techniques.

In Chapter Four, we show the decomposition of the price of a European call option as the difference of two tail expectations under the risk neutral measure and share measure. Provided that a closed form formulas of the cumulant generating function for the underlying asset price process under the two measures exist, the celebrated Lugannani-Rice formula can be applied to obtain the saddlepoint approximation for the European call option. This technique is extended to pricing European options under stochastic volatility and interest rate. We also show how to use saddlepoint approximation for pricing VIX futures and options under stochastic volatility model with jumps. The highlight of the chapter is the derivation of the saddlepoint approximation formulas for pricing options on discrete realized variance under the Levy processes and stochastic volatility processes with jumps.

In the last chapter, the saddlepoint approximation methods are applied to risk measures calculations in credit risk portfolios and pricing of the tranched Collateralized Debts Obligations. Two classes of default correlation models are considered in details, the CreditRisk$_+$ and Gaussian copula models. Risk measures are introduced in the axiomatic setting of coherent measure. Various saddlepoint approximation formulas are derived for calculating the Value-at-Risk and expected shortfall of the losses in credit portfolios, together with the computation of risk contributions of the obligors to these two risk measures. As an alternative to the computationally intensive recursive scheme to generate the loss distribution in the CreditRisk$_+$

model, efficient analytical saddlepoint approximations are derived. Illustrative numerical examples are presented to demonstrate numerical performance of different saddlepoint approximation formulas in computing risk measures of credit portfolios under the Vasicek one-factor default model. We also show how the calculations of the fair spread rates of the different tranches in Collaterized Debts Obligations can be effected by computing the tail expectations of the credit losses using appropriate saddlepoint approximation formulas.

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