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Mathematical Models of Financial Derivatives

Second Edition

Springer

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To my wife Oi Chun,
our two daughters Grace and Joyce

Preface

Objectives and Audience

In the past three decades, we have witnessed the phenomenal growth in the trading of financial derivatives and structured products in the financial markets around the globe and the surge in research on derivative pricing theory. Leading financial institutions are hiring graduates with a science background who can use advanced analytical and numerical techniques to price financial derivatives and manage portfolio risks, a phenomenon coined as *Rocket Science on Wall Street*. There are now more than a hundred Master level degreed programs in Financial Engineering/Quantitative Finance/Computational Finance in different continents. This book is written as an introductory textbook on derivative pricing theory for students enrolled in these degree programs. Another audience of the book may include practitioners in quantitative teams in financial institutions who would like to acquire the knowledge of option pricing techniques and explore the new development in pricing models of exotic structured derivatives. The level of mathematics in this book is tailored to readers with preparation at the advanced undergraduate level of science and engineering majors, in particular, basic proficiencies in probability and statistics, differential equations, numerical methods, and mathematical analysis. Advance knowledge in stochastic processes that are relevant to the martingale pricing theory, like stochastic differential calculus and theory of martingale, are introduced in this book.

The cornerstones of derivative pricing theory are the Black–Scholes–Merton pricing model and the martingale pricing theory of financial derivatives. The renowned risk neutral valuation principle states that the price of a derivative is given by the expectation of the discounted terminal payoff under the risk neutral measure, in accordance with the property that discounted security prices are martingales under this measure in the financial world of absence of arbitrage opportunities. This second edition presents a substantial revision of the first edition. The new edition presents the theory behind modeling derivatives, with a strong focus on the martingale pricing principle. The continuous time martingale pricing theory is motivated through the analysis of the underlying financial economics principles within a discrete time framework. A wide range of financial derivatives commonly traded in the equity and fixed income markets are analyzed, emphasizing on the aspects of pricing, hedging, and their risk management. Starting from the Black–Scholes–Merton formulation of the option pricing model, readers are guided through the book on the new advances in the state-of-the-art derivative pricing models and interest rate models. Both analytic techniques and numerical methods for solving various types of derivative pricing models are emphasized. A large collection of closed form price formulas of various exotic path dependent equity options (like barrier options, lookback options, Asian options, and American options) and fixed income derivatives are documented.

Guide to the Chapters

This book contains eight chapters, with each chapter being ended with a comprehensive set of well thought out exercises. These problems not only provide the stimulus for refreshing the concepts and knowledge acquired from the text, they also help lead the readers to new research results and concepts found scattered in recent journal articles on the pricing theory of financial derivatives.

The first chapter serves as an introduction to the basic derivative instruments, like the forward contracts, options, and swaps. Various definitions of terms in financial economics, say, self-financing strategy, arbitrage, hedging strategy are presented. We illustrate how to deduce the rational boundaries on option values without any distribution assumptions on the dynamics of the price of the underlying asset.

In Chap. 2, the theory of financial economics is used to show that the absence of arbitrage is equivalent to the existence of an equivalent martingale measure under the discrete securities models. This important result is coined as the Fundamental Theorem of Asset Pricing. This leads to the risk neutral valuation principle, which states that the price of an attainable contingent claim is given by the expectation of the discounted value of the claim under a risk neutral measure. The concepts of attainable contingent claims, absence of arbitrage and risk neutrality form the cornerstones of the modern option pricing theory. Brownian processes and basic analytic tools in stochastic calculus are introduced. In particular, we discuss the Feynman–Kac representation, Radon–Nikodym derivative between two probability measures and the Girsanov theorem that effects the change of measure on an Ito process.

Some of the highlights of the book appear in Chap. 3, where the Black–Scholes–Merton formulation of the option pricing model and the martingale pricing approach of financial derivatives are introduced. We illustrate how to apply the pricing theory to obtain the price formulas of different types of European options. Various extensions of the Black–Scholes–Merton framework are discussed, including the transaction costs model, jump-diffusion model, and stochastic volatility model.

Path dependent options are options with payoff structures that are related to the path history of the asset price process during the option's life. The common examples are the barrier options with the knock-out feature, the Asian options with the averaging feature, and the lookback options whose payoff depends on the realized extremum value of the asset price process. In Chap. 4, we derive the price formulas of the various types of European path dependent options under the Geometric Brownian process assumption of the underlying asset price.

Chapter 5 is concerned with the pricing of American options. We present the characterization of the optimal exercise boundary associated with the American option models. In particular, we examine the behavior of the exercise boundary before and after a discrete dividend payment, and immediately prior to expiry. The two common pricing formulations of the American options, the linear complementarity formulation and the optimal stopping formulation, are discussed. We show how to express the early exercise premium in terms of the exercise boundary in the form of an integral representation. Since analytic price formulas are in general not available for American options, we present several analytic approximation methods for pricing American options. We also consider the pricing models for the American barrier options, the Russian option and the reset-strike options.

Since option models which have closed price formulas are rare, it is common to resort to numerical methods for valuation of option prices. The usual numerical approaches in option valuation are the lattice tree methods, finite difference algorithms, and Monte Carlo simulation. The primary essence of the lattice tree methods is the simulation of the continuous asset price process by a discrete random walk model. The finite difference approach seeks the discretization of the differential operators in the Black–Scholes equation. The Monte Carlo simulation method provides a probabilistic solution to the option pricing problems by simulating the random process of the asset price. An account of option pricing algorithms using these approaches is presented in Chap. 6.

Chapter 7 deals with the characterization of the various interest rate models and pricing of bonds. We start our discussion with the class of one-factor short rate models, and extend to multi-factor models. The Heath–Jarrow–Morton (HJM) approach of modeling the stochastic movement of the forward rates is discussed. The HJM methodologies provide a uniform approach to modeling the instantaneous interest rates. We also present the formulation of the forward LIBOR (London-Inter-Bank-Offered-Rate) process under the Gaussian HJM framework.

The last chapter provides an exposition on the pricing models of several commonly traded interest rate derivatives, like the bond options, range notes, interest rate caps, and swaptions. To facilitate the pricing of equity derivatives under stochastic interest rates, the technique of the forward measure is introduced. Under the forward measure, the bond price is used as the numeraire. In the pricing of the class of LIBOR derivative products, it is more effective to use the LIBORs as the underlying state variables in the pricing models. To each forward LIBOR process, the Lognormal LIBOR model assigns a forward measure defined with respect to the settlement date of the forward rate. Unlike the HJM approach which is based on the non-observable instantaneous forward rates, the Lognormal LIBOR models are based on the observable market interest rates. Similarly, the pricing of a swaption can be effectively performed under the Lognormal Swap Rate model, where an annuity (sum of bond prices) is used as the numeraire in the appropriate swap measure. Lastly, we consider the hedging and pricing of cross-currency interest rate swaps under an appropriate two-currency LIBOR model.

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This book benefits greatly from the advice and comments from colleagues and students through the various dialogues in research seminars and classroom discussions. Some of materials used in the book are outgrowths from the new results in research publications that I have coauthored with colleagues and former Ph.D. students. Special thanks go to Lixin Wu, Min Dai, Hong Yu, Hoi Ying Wong, Ka Wo Lau, Seng Yuen Leung, Chi Chiu Chu, Kwai Sun Leung, and Jin Kong for their continuous research interaction and constructive comments on the book manuscript. Also, I would like to thank Ms. Odissa Wong for her careful typing and editing of the manuscript, and her patience in entertaining the seemingly endless changes in the process. Last but not least, sincere thanks go to my wife, Oi Chun and our two daughters, Grace and Joyce, for their forbearance while this book was being written. Their love and care have always been my source of support in everyday life and work.

Final Words on the Book Cover Design

One can find the Bank of China Tower in Hong Kong and the Hong Kong Legislative Council Building in the background underneath the usual yellow and blue colors on the book cover of this Springer text. The design serves as a compliment on the recent acute growth of the financial markets in Hong Kong, which benefits from the phenomenal economic development in China and the rule of law under the Hong Kong system.

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