Discussion on

VIX Option Valuation

presented by

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Scope of the study

- Derive a VIX (volatility index) option model that allows simultaneous correlated and state dependent jumps in stochastic volatility and SPX (S & P index) returns.

- Assess the extent of the misspecification of various forms of stochastic volatility models in pricing VIX derivatives.

Address the question: Can complex jump specifications add explanatory power in fitting options?

Specification of the stochastic volatility model

- Allows for simultaneous correlated and state dependent jumps in both stochastic volatility and returns.

- In the proposed SVSCJ model, the dynamics of the forward asset price under the forward measure Q^T is given by

$$\begin{aligned} \frac{dF_t}{F_t} &= \sqrt{v_t}\, dW_{S,t} + J_t\, dN_t - \lambda_t k\, dt \\ dv_t &= k_v(\theta_v - v_t)\, dt + \sigma_v \sqrt{v_t}\, dW_{v,t} + Z_v\, dN_t \end{aligned}$$

  where $J_t = \exp(z_S) - 1$ is the percentage price jump size with mean k. $W_{S,t}$ and $W_{v,t}$ are correlated Brownian processes with

$$\rho\, dt = \text{cov}(dW_{S,t}, dW_{v,t}).$$

The instantaneous variance v follows a mena-reverting square root process with exponentially distributed jump size z_v that is correlated to price jump size z_S by

$$z_S = \mu_j + \rho_j z_v.$$

Jumps in volatility are assumed to have an exponential distribution

$$z_v \sim \exp(\mu_v),$$

while jumps in asset log-prices are normally distributed conditional on the realization of z_v,

$$z_S | z_v \sim N(\mu_j + \rho_j z_v, \sigma_j^2).$$

- The underlying return and its volatility share the same jump arrival uncertainty followed by a Poisson process N_t.

Major results

- By adopting a replication strategy using a particular strip of SPX options, the forward price of total variance can be obtained in closed form.

- Closed form solutions to the fair value of the VIX futures and VIX options are obtained using the characteristic function of the log VIX squared under the forward probability measure Q^T.

- Examine the internal consistency of each stochastic volatility model's implied parameters with those implicit in the time series of the VIX futures prices.

- Two performance yardsticks show that a model with stochastic volaility and state dependent correlated jumps in SPX returns and volatility is a better alternative in terms of pricing.

Comments and queries

1. Full correlation of return and volatility jumps

   - In the dynamics of the forward price F_t under Q^T, the jumps in returns and volatility are compound Poisson processes $z_S\, dN_t$ and $z_v\, dN_t$ that are fully correlated with $z_S = \mu_j + \rho_j z_v$.
   - This correlation assumption leads to the relative ease in the analytic tractability of the VIX option formula.
   - This is because jump-related characteristic function contains all the information required to describe the joint behavior of jumps in the asset price and volatility.
   - Would this correlation assumption limit the generalization of the SPX price dynamics?

2. VIX futures price formulas

- The empirical studies used the time series data of the VIX futures price while analytic formulas for VIX forward prices are computed and adopted in the empirical analysis.
- This is based on the assumption of deterministic interest rate in their study so that forward prices and futures prices are the same. The assumption of deterministic interest rates (or zero correlation of index return and interest rate) is weakened for long dated derivatives.

Does there exist an alternative approach to compute the VIX futures price?

For example, Carr and Wu show that

$$
\begin{aligned}
E^Q[RV_{t,T}] \;=\; & E^Q\left[\frac{2}{T-t}\left[\int_0^{F_t}\frac{1}{K^2}(K-S_T)^+\,dK+\int_{F_t}^{\infty}\frac{1}{K^2}(S_T-K)^+\,dK\right.\right.\\
& \left.\left.+\;\int_t^T\left(\frac{1}{F_s}-\frac{1}{F_t}\right)\,dF_s\right]\right]\\
& -\;E^Q\left[\frac{2}{T-t}\int_t^T\int_0^{\infty}\left(e^x-1-x-\frac{x^2}{2}\right)\mu(dx,ds)\right]
\end{aligned}
$$

where $\mu(dx, dt)$ is a random measure that counts the number of jumps of size $e^x - 1$ in the index price at time t.

3. Semi-negative results

The importance of inclusion of jumps in volatility is emphasized in the study. Jumps in volatility are said to provide a rapidly moving but persistent factor driving volatility. Unfortunately, the proposed SVSCJ model still exhibits significant structural misspecifications. All four stochastic volatility models rely on implausible levels of volatility variation of forward VIX to rationalize the observed option prices.

Does the addition of more jumps provide the clue to minimize model inconsistencies?