# Five topics of the final presentation for Math 6050E: PDEs, Fall 2016

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**Date: 9 December 2016**:

i) 9:00-12:00 Room 3494 near Lift 25-26.
ii) 14:00-18:00 Room 5504 near Lift 25-26.

## 1 Best constant in the Sobolev inequality

Let $n \geq 3$. Prove that there exists an optimal constant $C > 0$ such that

$$\|u\|_{L^{\frac{2n}{n-2}}(\mathbb{R}^n)} \leq C\|\nabla u\|_{L^2(\mathbb{R}^n)} \quad \text{for all } u \in C_c^\infty(\mathbb{R}^n).$$

Moreover, find this optimal constant explicitly.

Present any proof you like, not limited to the following references:

1. G. Talenti, *Best constant in Sobolev inequality*, Ann. Mat. Pura Appl. 110 (1976) 353–372.
2. T. Aubin, *Problemes isopérimétriques et espaces de Sobolev*, J. Differential Geometry 11 (4) (1976) 573–598.
3. Elliott H Lieb and Michael Loss, *Analysis*. Graduate Studies in Mathematics, 14. American Mathematical Society, Providence, RI, 1997.

## 2 BMO and John-Nirenberg's inequality

Present the following paper:
F. John and L. Nirenberg: *On functions of bounded mean oscillation*. Comm. Pure Appl. Math. 14 (1961) 415–426.

## 3 Moving plane method

Present the statement and proof of Theorem 1 in the following paper:
James Serrin, *A symmetry problem in potential theory*. Arch. Rational Mech. Anal. 43 (1971), 304–318.

# 4 Fractional Sobolev inequality

Present the statement and proof of Theorem 6.5 in the following paper:
Eleonora Di Nezza, Giampiero Palatucci and Enrico Valdinoci: *Hitchhiker's guide to the fractional Sobolev spaces*. Bull. Sci. Math. 136 (2012), no. 5, 521–573.

# 5 Two Gagliardo's theorems on Trace embedding: $W^{1,p}(\mathbb{R}^n) \hookrightarrow W^{1-\frac{1}{p},p}(\mathbb{R}^{n-1})$ and $trW^{1,1} = L^1$

Present the statements and proofs of Lemma 26, Theorem 25 and Theorem 26 in Chapter 11 of the paper:
Petru Mironescu, *Fine properties of functions: an introduction*.
Available at: https://cel.archives-ouvertes.fr/cel-00747696/document

See also

Petru Mironescu, *Note on Gagliardos theorem "$trW^{1,1} = L^1$"*.
Available at: https://hal.inria.fr/hal-01131162/document